FINAL REPORT

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Selected Reliability Studies for the NERVA Program

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Gentlemen:

As required by the provisions for research grants funded by the National Aeronautics and Space Administration, we are enclosing five copies of a Final Report on the above mentioned grant.

Yours truly,

Gerald R. Murphy

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Coordinator of Research Grants

GRM:ah

Enclosures (5)

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Abstract

An investigation was made into certain methods of reliability analysis that are particularly suitable for complex mechanisms, or systems, in which there are many interactions (electrical, chemical, mechanical, etc.). The methods developed here were intended to assist in the design of such mechanisms, especially for analysis of failure sensitivity to parameter variations and for estimating reliability where extensive and meaningful life-testing is not feasible. The system is modeled by a network of interconnected nodes. Each node is a state or mode of operation, or is an input or output node,and the branches (interconnection paths) are interactions (in many cases, failure mechanisms). Each interaction has a probability and a time distribution. An interactive matrix is formed, the rows and columns of which are the nodes. The probability of going from one node to another, and the number of m-element paths between the nodes, is calculated by application of matrix multiplication and summation. Conventional failure analyses cannot as easily handle these interactive problems. The network, with its probabilistic and time-dependent paths is also analyzed for reliability and failure nodes by a Monte Carlo, GERTS computer-simulation of system performance. Applications of these methods are made primarily to a high pressure, constrained bellows for the pump discharge line of a NERVA rocket engine, and secondarily to a railroad car wheel.

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I. Introduction

NASA's missions require sophisticated equipment of extraordinarily high reliability. The equipment functions are complex, yet the equipment must be of minimum size and weight, and often must operate in unusual environments. Since the missions are relatively few in number, expensive, and may involve human life, equipment failure can be disasterous.

Component or system failure almost always begins with the "failure", or abnormal state, of a material. This abnormal state may arise in a number of ways: inadequate design (misuse of materials), overstressing (mechanical thermal, electrical), poorly executed fabrication, or deficient inspection (either in practice or in principle).

Achievement of high reliability, and good estimates of reliability, depend upon a knowledge of the response of materials to their environment, especially with respect to the creation of abnormal (undesirable) states in the materials. Failure data exist for specific materials in certain known failure modes, and design studies take these particular failure states into account, often in an arbitrary manner.

Many books and articles have been published dealing with the estimation of reliability. A common method for determining reliability is to test the component or system under service conditions in statistically significant quantities and for a long enough time to accumulate date for estimating failure rates at an agreed-upon confidence level. The problem often is the necessity for lengthy times, and so one usually resorts to accelerated life tests that are based upon a known or reasonable correlation with normal service conditions. In many cases, however, it is not feasible to test significant quantities of the component or system. Here, then, one is thrown back to estimation schemes. These cannot be better than the input data or the ability of the engineer to identify and quantify the many failure possibilities. The engineer has two general reliability analysis schemes to help him find a reliability function: fault free analysis, and failure mode analysis[1]. In either scheme he will use empirical data and a knowledge of the physics of failure to identify various failure mechanisms and their dependence on environmental variables. The critical assumption usually made in these schemes is the independence of the failures of the component units that make up the system.

The purpose of the present work was to develop a method of reliability analysis which could be applied to a complex mechanism in which there were many electrical, mechanical and chemical interactions. In particular, the mechanism of primary interest was a component of the NERVA nuclear rocket. Traditional mathematical reliability analysis fails for such an environment because of the complexity of the interactions, while life testing is infeasible. The reliability techniques developed in this work were intended to assist in the design of such complex mechanisms, especially for sensitivity analysis in the initial design and for estimating the reliability in the final design.

These techniques can also lead to both the discovery and quantification of failure mechanisms by an in-depth examination of a specific component and its interaction with the environment, emphasis being placed on the materials' states and behavior with time.

The analysis consists of a thorough examination of the response of the component materials to their environment. Response and environment are interpreted in a broad sense. Response is here meant to be an unusual or abnormal state of the material. This state may or may not be reversible or permanent. It need not be a failure state in itself, but it could lead to "failure" of the material or of another material, or of the component. Environment is the totality of parameters acting upon the material, and include those of chemical, mechanical, heat, and nuclear origin. In certain cases, electric or magnetic fields might also be included.

The examination of response to environment is, therefore, a study of material interactions. The interactions will include feedback and cyclical effects, processes involving several steps (or mechanisms), and synergistic processes (i.e., simultaneous interactions producing a response that is far greater than either interaction operating alone). Many of the interactions will be those usually classed as failure mechanisms, that is, interactions leading directly to catastrophic failure. However, other interactions will lead to more subtle, less direct failure possibilities involving material changes that, in time, will produce failure. Note that the environment itself may be modified by certain material changes; for example, a change in elastic modulus of one material could alter the "environment" forces or frequency of forces on another material.

As an aid in the examination we will use methods such as GERT, Monte Carlo simulation for analysis of a network representing the interactive mechanisms and failure modes of the component or system. This network will also help identify less obvious or complicated failure modes and sequences.

II. Description of NERVA Component

The reliability techniques investigated in this program were applied to a possible component of a NERVA engine. This component, selected by NASA, was a high pressure, internally restrained bellows to be used in the pump discharge line. A sketch of the component is shown in Fig. 1. The flexibility of the bellows provides for articulation of the engine with respect to the rocket, and the flow of liquid hydrogen propellant from pump to reactor takes place through the bellows. Two internal tripods supporting a ball and socket, respectively, prevent extension of the bellows.

This component must operate in both earth and space environments, at temperatures of -423°F, and at hydrogen pressures of 1040 psia. Calculations of the stresses that would occur on this component had been made by engineers at the Aerojet-General Corporation, Sacramento, California. It was also known that the hydrogen stream would carry with it a quantity of particulate matter. A useful operating life of ten hours was expected intermittently for 60 cycles over about a month.

For the purposes of this investigation a knowledge of the precise operating stresses and environmental conditions were not necessary.

III. Analytic Techniques

In devices as complex as a Nerva Rocket components of the system can often operate in one of several states or modes, one or several of which may effect the reliability of other elements. As an example, consider a support rib in a group of three which are supporting another element (Fig. 1). If any one of the ribs should yield, the device itself does not fail but the new mode of operation (one support rib having yielded) will now have an effect on the reliability of the entire device. Analyzing the reliability of the system by conventional reliability mathematics is not feasible either in the early stages of the design or in the later stages of design (when more information is available from the system's performance). In our approach a complete enumeration was made of all components in the system and their various modes of operation. Next all exogenous variables which could affect the reliability of this system were enumerated. Examples of such variables are heat, vibrations, moisture, foreign particles, etc. After listing the modes of operation and the exogenous variables, a matrix was constructed which indicated whether any particular mode of operation could cause some other component and/or itself to operate in a new mode at a later time. This matrix is somewhat similar to the classical "fault-tree" analysis (fault trees, however, are more graphic than analytic and do not show possible feedback). This matrix was then raised to higher and higher powers until the product finally vanished. The elements of the matrix that is formed from the sum of all of the product matrices indicate the number of cause-effect paths from any other single mode of operation.

In addition, the original matrix also served as the basis for a Monte Carlo simulation. In the simulation, account was taken of the probability that any component will change its mode of operation as well as the time it will take for this change of mode to occur. The simulation was carried out using the GERTS program which was developed for NASA in 1968.[2]

GERTS is a general simulation program for stochastic networks. The branches or arcs of the network correspond to activities or processes and the nodes of the network correspond to the end of one or a set of activities and the beginning of other activities. The reliability of a system can be modeled as a network wherein the interaction (possible failure) mechanisms are represented as branches, and nodes represent the modes of operation (or states) of the system upon the completion of an interaction mechanism. Each branch is characterized by the probability that the mechanism will be set into motion and the probability distribution of the length of time until the mechanism is completed, or has possibly reached a point at which a new mode of operation occurs. The network contains two special nodes: sink nodes and source nodes. The source nodes represent both the initial state of the system and the environmental factors which give rise to failure mechanisms. The sink nodes represent the events of mission success or mission failure. A more complete and detailed description of GERTS is contained in Appendix A.

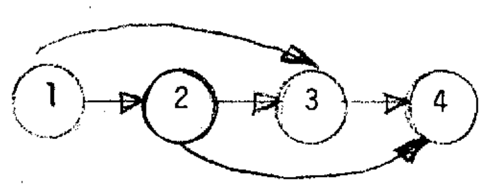
Application of Reliability Analysis to NERVA Bellows

As a vehicle for demonstrating the reliability analysis techniques developed in this research, a constrained bellows for the nuclear rocket was selected (Fig. 1). Table 1a lists the components of the mechanism. Table 1b lists various conditions and Table 1c lists pertinent exogenous variables. The matrix of component states and exogenous variables is 59 by 59. A component state represents the combination of a component plus a condition to give a node in the graphical network; thus, 2A represents an inlet support ring in the yielded condition. Table 2a lists the nodes (row or columns). All the matrix elements (interactions) are zero except those listed in Table 2b.

A simpler version of the bellows was also considered which allows closer observation of the reliability techniques developed. Figure 2 shows graphically the relationship between the variables in this smaller system. The relationships between modes of operation and exogenous variables can be represented by an n x n matrix. If there is a direct one step cause-effect interaction betwen two variables or modes of operation a one is shown, otherwise a zero. That is:

$$a_{ij} = 1 \quad \text{if state } i \text{ can lead directly in one step to state } j.$$
$$= 0 \quad \text{otherwise.}$$

This matrix has the property that when it is raised to the m^{th} power the elements show the number of paths between i and j containing exactly m branches. Consider the 4 state system:



$$
A = \begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 0 & 1 & 1 & 0 \\
2 & 0 & 0 & 1 & 1 \\
3 & 0 & 0 & 0 & 1 \\
4 & 0 & 0 & 0 & 0
\end{array}
\qquad
A^2 = \begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 0 & 0 & 1 & 2 \\
2 & 0 & 0 & 0 & 1 \\
3 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0
\end{array}
$$

$$
A^3 = \begin{array}{cccc}
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}
\qquad
A^4 = \begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}
$$

Thus, there is one 2-branch path between states 1 and 3, one 2-branch path between states 2 and 4, two 2-branch paths between states 1 and 4 and oen 3-branch path between states 1 and 4. If we weight the respective branches by the probabilities that each may be taken we can repeat the above procedure and will nowhave the probability any given path is completed. This assumes, of course, that the paths are independent. Figure 3 shows the matrix of relationships for the small system considered. This maxtrix was raised to the seventh power and all terms vanished. The product matrices were summed and are shown in Fig. 4. From this figure it can be seen that rib yielding can lead to inlet support ring cracking by three separate paths. This indicates that there is high possibility of interaction between the rib yield state and the inlet-support ring crack. The matrices can also be used to examine the lengths of the paths. Further, a probability for each interaction was inserted in place of the original element 0 or 1, and the elements of the product matrix then gave the probability of a given interaction between two states. In order to incorporate the effect of time in the failure analysis a time distribution was assigned to each interaction.

A Monte Carlo, GERT, simulation was performed on this smaller matrix and the results of 5000 mission simulations appear in Table 3. Table 3 shows the probability of failure nodes (2C, 17C and 17D) being realized along with the relative time before failure. The time required for a state to change to another state was estimated to be either instantaneous, intermediate (uniformly distributed between 1 and 5) or long (uniformly distribued between 5 and 10). These same distributions were subsequently used in the GERT simulation of the large network. It was assumed that the length of the mission was 10. The probability that a state would change was assumed to be either .05 or .005. These estimates were subsequently used in the GERT simulation of the large network. Based on the above probabilities and times, the reliability of the bellows was .9688. Actual data would need to be gathered experimentally or by physics-of-failure models. This particular analysis is for illustration only.

The analysis of the small matrix version of the bellows points out several interesting characteristics of the system reliability. Looking first at the "Total Matrix," Fig. 4, it can be seen that there is a large number (32) of paths which can lead to state 2C - inlet support ring fractured. At the same time it can be seen that there are only 9 paths which will lead to state 17C - bellows fracture. This would indicate that the bellows is less likely to fail from a rupture than it is from an inlet ring fracture. When each branch is weighted by the probability of the associated failure mechanism the probability of an inlet ring fracture is approximately .001 while the probability of a bellows rupture is .00007. When the time character of the failure mechanism is introduced and a GERT simulation is performed on the same network, the bellows failed 4 times in 5000 simulations due to an inlet ring support fracture and failed 52 times due to a bellows fracture (see Table 3). Here it may be seen that these models provide a powerful tool for determining the elements in the system which are most likely to cause failure of the system.

A similar analysis was performed on the larger matrix. The results of the simulation analysis are shown in Table 4. The difference between the simulation analysis and the matrix analysis is primarily due to incorporation of the distribution in time for the interaction mechanisms. When the matrix analysis of the large system was performed (without probabilities) the number of paths betwen external causes and failures became alarmingly large. It was not determined whether this was due to a truly large number of paths or to a topological inconsistency in the network or to a bona fide loop. A loop could lead to positive feed back and thus to sustained oscillations of the interactive mechanisms. This is an important part of the matrix analysis but the scope of this work did not permit the analysis of techniques for detecting loops. In the GERT simulation of the large system it became apparent that the system would most likely fail because of a bellows rupture (Table 4). The mechanisms which lead to a bellows rupture were traced in the simulation and are shown in Table 5. The analysis suggests that protection of the bellows is an important design consideration.

IV. Application to Other Problems

A small investigation was made into the application of the concepts and methods described above to another mechanical system. A system was sought which was simple and yet possessed the characteristics of interaction among system variables and multiple failure modes. The railroad wheel was chosen because of recent renewed interest in railway safety and because the complexity of the service conditions of railroad wheels made them an attractive candidate.

There are numerous reports in the literature of work related to isolated aspects of railroad wheel service conditions and wheel failures. But there has been no work reported on the effect of combined service conditions on all failure modes. Novak and Eck[3] have calculated the wheel stresses caused by combined simulated thermal and mechanical service loads. Bruner, et al[4] have reported the effects of design variation and service stresses in railroad wheels and an analysis of the residual thermal and loading stresses together with their relation to fatigue damage. A study of the tread temperature during braking in grade operation has been reported by Cabble[5]. Measurement and analysis of the lateral, vertical and contact wheel - rail forces has been performed by Martin and Hay[6] and Peterson, Freeman and Wandrisco[7]. The information contained in these studies has been useful in our effort to construct an overall loading - response picture of the wheel.

Since the objective of the study of the wheel was to demonstrate the applicability of the failure mode analysis described previously to a different mechanical system it was decided to make the system as simple as possible without sacrificing the interactive features. To reduce the number of components in the system the wheel was studied without consideration of the axle and the bearings. The terminology used to describe components and failure modes was taken from the Wheel and Axle Manual and the Manual of Standards and Recommended Practices of the American Association of Railroads.

Only the surface (and immediate subsurface) of the wheel rim was considered. The interactive matrix describing this system is discussed below.

The material properties of the wheel that were considered are yield strength, Brinell Hardness and fatigue limit. The service loading was attributed to the mechanical loading (car load, track bed, rocking) and to the temperature changes resulting from braking. These service inputs are then expressed as a stress state which, when combined with material properties, produce mechanical effects such as elastic and plastic deformation, crack growth and wear. The thermal history causes, in addition, annealing effects which show up in the magnitude of yield stress and hardness. An additional effect of the deformation of the wheel is a residual stress field which must be continuously added to the loading and thermal stresses.

When the radial crack growth resulting from the net circumferential tension stress exceeds a critical value, the failure state of <u>thermal cracking</u> is reached. When the plastic flow because of high shear stress and low yield stress becomes excessive, the failure state of <u>built-up tread</u> is obtained. And when the subsurface circumferential crack length exceeds a critical value the failure state of <u>shelled tread</u> is reached. The interactions of these variables are shown in matrix form in Fig. 5. The columns are considered to be the causes and the rows to be the effects. The zero elements of the matrix indicate no interaction and the unitary elements denote an interaction between system variables.

Time did not permit further elaboration of this model; however, the following paragraphs indicate the approach that would have been taken.

The simulation scheme was to have involved simultaneous equations in which the matrix elements were coefficients relating variables (independent). The coefficients may be constant, or functions selected to model the physical interaction. For example, the fourth row of the matrix indicates:

$$H = H(T,\sigma)$$

Many of these functions, such as the relation between loading situations and the stress in the wheel, would be in tabular form for the computer. The sourcesof these functions come both from our understanding of the specific interaction and from data or analysis in the literature.

Computer simulation would then be conducted by programming the stress (load) and temperature so as to simulate typical service conditions. References (3)and (7) would have been helpful in this programming. The equations were to be solved at regular time intervals. The values of the <u>row</u> variables obtained from the first interval solutions would be used as the <u>column</u> variables for the second interval, etc.

V. Conclusions

The physical state or mode of operation of a complex system, or component, having interdependent changes may be modeled by a network of interconnected nodes. Each node is a state or mode of operation, or is an input or output node, and the branches (interconnection paths) are interactions (in many cases, failure mechanisms). Each interaction has a probability and a time distribution. An interactive matrix may be formed, the rows and columns of which are the modes. The probability of going from one node to another may be calculated by application of matrix multiplication. The elements of the matrix which is the sum of the product matrices are either the number of paths between the nodes or the probability of occurrence of that interaction. Conventional failure-mode or fault tree analyses cannot easily handle the interactive problem, whereas the matrix method presented here can do so quantitatively and easily.

The network, with its probabilistic and time-dependent paths, can also be used for a Monte Carlo, GERTS, computer-simulation of system performance. This simulation can produce reliability estimates, can show the probabilities of the various failure modes, and can show the sensitivity of the reliability and failure modes to changes in probability and time functions. This last feature is especially valuable when insufficient data is available for some of the interactions.

Although the above methods (matrix multiplication and probabilistic

Monte Carlo simulation) are more demanding of knowledge about materials, system, and failure-mechanisms than conventional ways for estimating reliability, the above methods yield more information. They also lead the engineer into a deeper analysis of the system, which may result in discovery of new failure modes, especially for interactive systems, and a consequent greater reliability.

VI. References

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TABLE 1a

Components for the Constrained Bellows

1. inlet pipe
2. inlet support ring
3. rib weld to inlet support ring
4. rib
5. rib weld to socket support
6. socket support
7. socket
8. ball-socket interface
9. ball
10. ball support
11. rib weld to ball support
12. rib
13. rib weld to outlet support ring
14. outlet support ring
15. outlet pipe
16. bellows weld to inlet support
17. bellows
18. bellows weld to outlet support

TABLE 1b

Conditions for the Constrained Bellows

A. yield
B. crack
C. fracture (broken)
D. rupture
E. dent
F. gall
G. wear
H. cavitation
I. welded
J. flow obstruction
K. good
L. loose

TABLE 1c

Inputs for the Constrained Bellows

19. vibration, external
20. vibration, flow
21. pressure LH_2
22. pressure variations LH_2
23. temperature
24. temperature variations
25. radiation
26. LH_2
27. material microstructure
28. material composition
29. weld defect
30. improper assembly
31. projectile, internal (large and small)
32. projectile, external
33. vacuum
34. air
35. moisture
36. normal operating stresses
37. angulation
38. abnormal stresses
39. starting shock
40. stopping shock

TABLE 2A

Nodes (Row & Columns) Of Large Matrix

NODE	DESCRIPTION
20	Vibration External
21	Vibration - Flow
22	Pressure LHZ
23	Pressure Var. LHZ
25	Temperature Var.
31	Improper Assembly
32	Projectile, Internal
33	Projectile, External
38	Angulation
39	Abnormal Stress
40	Starting Shock
41	Stopping Shock
42	Cavitation
2A	Inlet Support Ring: Yielded
2B	Inlet Support Ring: Cracked
2C	Inlet Support Ring: Fractured (failure)
2E	Inlet Support Ring: Dented
2J	Inlet Support Ring: Flow Obstruction
2K	Inlet Support Ring: Good
3A	Rib Weld: Yielded
3B	Rib Weld: Cracked
3C	Rib Weld: Fractured
3K	Rib Weld: Good
4A	Rib: Yielded
4B	Rib: Cracked
4C	Rib: Fractured
4H	Rib: Cavitation
4K	Rib: Good
4L	Rib: Loose
5A	Rib Weld to Support Ring: Yielded
5B	Rib Weld to Support Ring: Cracked
5C	Rib Weld to Support Ring: Fractured
5K	Rib Weld to Support Ring: Good
6A	Socket Support: Yielded
6B	Socket Support: Cracked
6C	Socket Support: Fractured
6H	Socket Support: Cavitation
6K	Socket Support: Good
7A	Socket: Yielded
7B	Socket: Cracked
7C	Socket: Fractured
7F	Socket: Galled
7G	Socket: Wear
7K	Socket: Good
7L	Socket: Loose
8I	Ball-Socket Interface: Welded (failure)
8K	Ball-Socket Interface: Good

TABLE 2A (Continued)

NODE	DESCRIPTION
16A	Bellows Weld: Yielded
16B	Bellows Weld: Cracked
16C	Bellows Weld: Fractured (failure)
16K	Bellows Weld: Good
17A	Bellows: Yielded
17B	Bellows: Cracked
17C	Bellows: Fractured (failure)
17D	Bellows: Ruptured (failure)
17E	Bellows: Dented
17J	Bellows: Flow Obstructed
17K	Bellows: Good

TABLE 2B

Probability Of Occurrence And Time Distribution
Of Nonzero Elements Of Large Matrix

START NODE	END NODE	PROBABILITY OF OCCURRENCE	TIME FOR* OCCURRENCE
23	2A	.005	1
32	2A	.005	1
33	2A	.005	1
39	2A	.005	1
3C	2A	.005	3
4C	2A	.005	3
4H	2A	.005	3
4L	2A	.005	3
5C	2A	.005	3
6C	2A	.005	3
8I	2A	.005	1
20	2B	.005	2
21	2B	.005	2
22	2B	.005	1
23	2B	.005	1
25	2B	.005	2
32	2B	.005	1
33	2B	.005	1
39	2B	.005	1
40	2B	.005	1
41	2B	.005	1
2A	2B	.050	1
2E	2B	.050	1
2J	2B	.005	1
3A	2B	.005	1
3C	2B	.005	1
4A	2B	.005	1
4C	2B	.005	1
4H	2B	.005	1
4L	2B	.005	1
5C	2B	.005	1
8I	2B	.005	1
2B	2C	.050	2
32	2E	.005	1
33	2E	.005	1
32	2J	.005	1
33	2J	.005	1
2E	2J	.005	1

* 1 implies instantaneous
2 implies between 0 and 5
3 implies between 0 and 10

TABLE 2B (Continued)

START NODE	END NODE	PROBABILITY OF OCCURRENCE	TIME FOR* OCCURRENCE
22	3A	.005	1
23	3A	.005	1
32	3A	.005	1
39	3A	.005	1
40	3A	.005	1
41	3A	.005	1
4C	3A	.005	3
4H	3A	.005	3
4L	3A	.005	3
5C	3A	.005	3
6C	3A	.005	3
8I	3A	.005	1
20	3B	.005	2
21	3B	.005	2
22	3B	.005	1
23	3B	.005	1
25	3B	.005	2
32	3B	.005	1
39	3B	.005	1
40	3B	.005	1
41	3B	.005	1
42	3B	.005	2
2A	3B	.050	1
2E	3B	.050	1
2J	3B	.005	1
3A	3B	.005	1
4A	3B	.005	1
4C	3B	.005	1
4H	3B	.005	1
4L	3B	.005	1
8I	3B	.005	1
3B	3C	.050	2
22	4A	.005	1
23	4A	.005	1
31	4A	.005	1
32	4A	.005	1
39	4A	.005	1
40	4A	.005	1
41	4A	.005	1
42	4A	.005	1
2J	4A	.005	1
3C	4A	.500	1
4C	4A	.500	1
4H	4A	.050	1
4L	4A	.500	1
5C	4A	.500	1
6C	4A	.500	1
8I	4A	.005	1

TABLE 2B (Continued)

START NODE	END NODE	PROBABILITY OF OCCURRENCE	TIME FOR* OCCURRENCE
4B	4C	.050	2
31	4H	.005	1
3C	4H	.500	1
4C	4H	.500	1
5C	4H	.500	1
3C	4L	.005	1
4C	4L	.005	1
5C	4L	.005	1
22	5A	.005	1
23	5A	.005	1
32	5A	.005	1
39	5A	.005	1
40	5A	.005	1
41	5A	.005	1
4C	5A	.500	1
4H	5A	.050	1
4L	5A	.500	1
5C	5A	.500	1
6C	5A	.500	1
8I	5A	.005	1
20	5B	.005	2
21	5B	.005	2
22	5B	.005	1
23	5B	.005	1
25	5B	.005	2
32	5B	.005	1
40	5B	.005	1
41	5B	.005	1
42	5B	.005	2
2A	5B	.050	1
2J	5B	.005	1
3A	5B	.050	1
4A	5B	.050	1
4C	5B	.500	1
4H	5B	.050	1
4L	5B	.500	1
5A	5B	.050	1
8I	5B	.005	1
5B	5C	.050	2
22	6A	.005	1
23	6A	.005	1
31	6A	.005	1
32	6A	.005	1
39	6A	.005	1
40	6A	.005	1
41	6A	.005	1
8I	6A	.005	1

TABLE 2B (Continued)

START NODE	END NODE	PROBABILITY OF OCCURRENCE	TIME FOR* OCCURRENCE
20	4B	.005	2
21	4B	.005	2
22	4B	.005	2
23	4B	.005	2
25	4B	.005	2
32	4B	.005	1
39	4B	.005	1
40	4B	.005	1
41	4B	.005	1
42	4B	.005	2
2J	4B	.005	2
3C	4B	.050	1
4A	4B	.050	1
4C	4B	.050	1
4H	4B	.005	1
4L	4B	.050	1
5C	4B	.050	1
8I	4B	.005	1
20	6B	.005	2
21	6B	.005	2
22	6B	.005	1
23	6B	.005	1
25	6B	.005	2
31	6B	.005	1
32	6B	.005	1
39	6B	.005	1
40	6B	.005	1
41	6B	.005	1
42	6B	.005	2
6A	6B	.050	1
8I	6B	.005	1
6B	6C	.050	2
31	6H	.005	1
4H	6H	.500	1
6C	6H	.050	1
22	7A	.005	1
23	7A	.005	1
31	7A	.005	1
32	7A	.005	1
39	7A	.005	1
40	7A	.005	1
41	7A	.005	1
8I	7A	.005	1
20	7B	.005	2
21	7B	.005	2
22	7B	.005	1
23	7B	.005	1
25	7B	.005	2

TABLE 2B (Continued)

START NODE	END NODE	PROBABILITY OF OCCURRENCE	TIME FOR* OCCURRENCE
31	7B	.005	1
32	7B	.005	1
39	7B	.005	1
41	7B	.005	1
42	7B	.005	2
6A	7B	.050	1
7A	7B	.050	1
8I	7B	.005	1
7B	7C	.050	2
32	7F	.005	3
38	7F	.005	3
40	7F	.005	1
41	7F	.005	1
3C	7F	.050	3
4C	7F	.050	3
4H	7F	.050	3
4L	7F	.050	3
5C	7F	.050	3
7A	7F	.050	3
7B	7F	.050	3
7C	7F	.050	3
7G	7F	.050	3
32	7G	.005	3
38	7G	.005	3
40	7G	.005	1
41	7G	.005	1
3C	7G	.050	3
4C	7G	.050	3
4H	7G	.050	3
4L	7G	.050	3
5C	7G	.050	3
7A	7G	.050	3
7B	7G	.050	3
7C	7G	.050	3
32	8I	.005	1
7F	8I	.050	3
32	16A	.005	1
33	16A	.005	1
39	16A	.005	1
40	16A	.005	1
41	16A	.005	1
2A	16A	.005	1
2E	16A	.005	1
2J	16A	.005	1
20	16B	.005	2
21	16B	.005	2
22	16B	.005	1

TABLE 2B (Continued)

START NODE	END NODE	PROBABILITY OF OCCURRENCE	TIME FOR* OCCURRENCE
23	16B	.005	1
25	16B	.005	2
32	16B	.005	1
33	16B	.005	1
39	16B	.005	1
40	16B	.005	1
41	16B	.005	1
42	16B	.005	2
2A	16B	.050	1
2E	16B	.050	1
2J	16B	.005	1
16A	16B	.050	1
16B	16C	.050	2
22	17A	.005	1
23	17A	.005	1
31	17A	.005	1
32	17A	.005	1
33	17A	.005	1
38	17A	.005	1
39	17A	.005	1
40	17A	.005	1
41	17A	.005	1
4H	17A	.005	1
4L	17A	.005	1
20	17B	.005	3
21	17B	.005	3
22	17B	.005	1
23	17B	.005	1
25	17B	.005	2
32	17B	.005	1
33	17B	.005	1
39	17B	.005	1
40	17B	.005	1
41	17B	.005	1
42	17B	.005	2
4H	17B	.050	3
4L	17B	.050	3
17A	17B	.050	1
17B	17C	.050	2
23	17D	.005	1
32	17D	.005	1
33	17D	.005	1
4H	17D	.050	1
4L	17D	.050	1
32	17E	.005	1
33	17E	.005	1
4H	17E	.050	1
4L	17E	.050	1

TABLE 2B (Continued)

START NODE	END NODE	PROBABILITY OF OCCURRENCE	TIME FOR* OCCURRENCE
32	17J	.005	1
4H	17J	.050	1
4L	17J	.050	1
17E	17J	.005	1
32	21	.005	1
2E	21	.500	1
2J	21	.500	1
4H	21	.500	1
4L	21	.500	1
17E	21	.500	1
17J	21	.500	1
6H	21	.500	1
3C	32	.050	1
4C	32	.050	1
4L	32	.005	1
5C	32	.050	1
6C	32	.050	1
7C	32	.050	1
7F	32	.005	1
32	23	.005	1
2E	23	.050	1
2J	23	.500	1
4H	23	.500	1
4L	23	.500	1
17E	23	.050	1
17J	23	.500	1
6H	23	.500	1

Table 3

Results of 5000 GERTS Mission Simulations

Using Small Matrix for Constrained Bellows

Failure Node	Failure Prob.	Mean Time to Node	Standard deviation	Min. time	Max. time
2C	.0008	3.4759	15.037	1.38	4.627
17C	.0200	2.278	3.1049	0	9.9056
17D	.0104	0.0	.0	0	0

Table 4

Results of 1000 Mission GERT Simulations Using Large Matrix for Constrained Bellows.

Node	Prob.	Mean Time to Node	Std. Dev.	Min. Time	Max. Time
Inlet Ring Fract.	.002	4.4	3.8	1.7	7.1
Ball-Socket Interface Weld.	No values recorded.				
Bellows Weld Fracture	No values recorded.				
Bellows Fracture	.001	5.4	0	5.4	5.4
Bellows Rupture	.008	0	0	0	0
Success	.989	10.0	0	10.0	10.0

Time = 10 = Success

Table 5

Nodes (States) Leading to Bellows Rupture (17D, Node 67)

Node #	Item Description	Item #	Freq.*
5	LH2 Press. Var.	23	16
14	Proj.-Int.	32	2
15	Proj.-Ext.	33	1000 (source)
39	Rib-displaced + I	4H + I	13
41	Rib-loose + I	4L + I	0

*from 1000 simulations GERT run.

Possible conclusions:

(1) shield bellows

(2) examine more closely the possibilities for LH2 pressure variations and rib displacements.

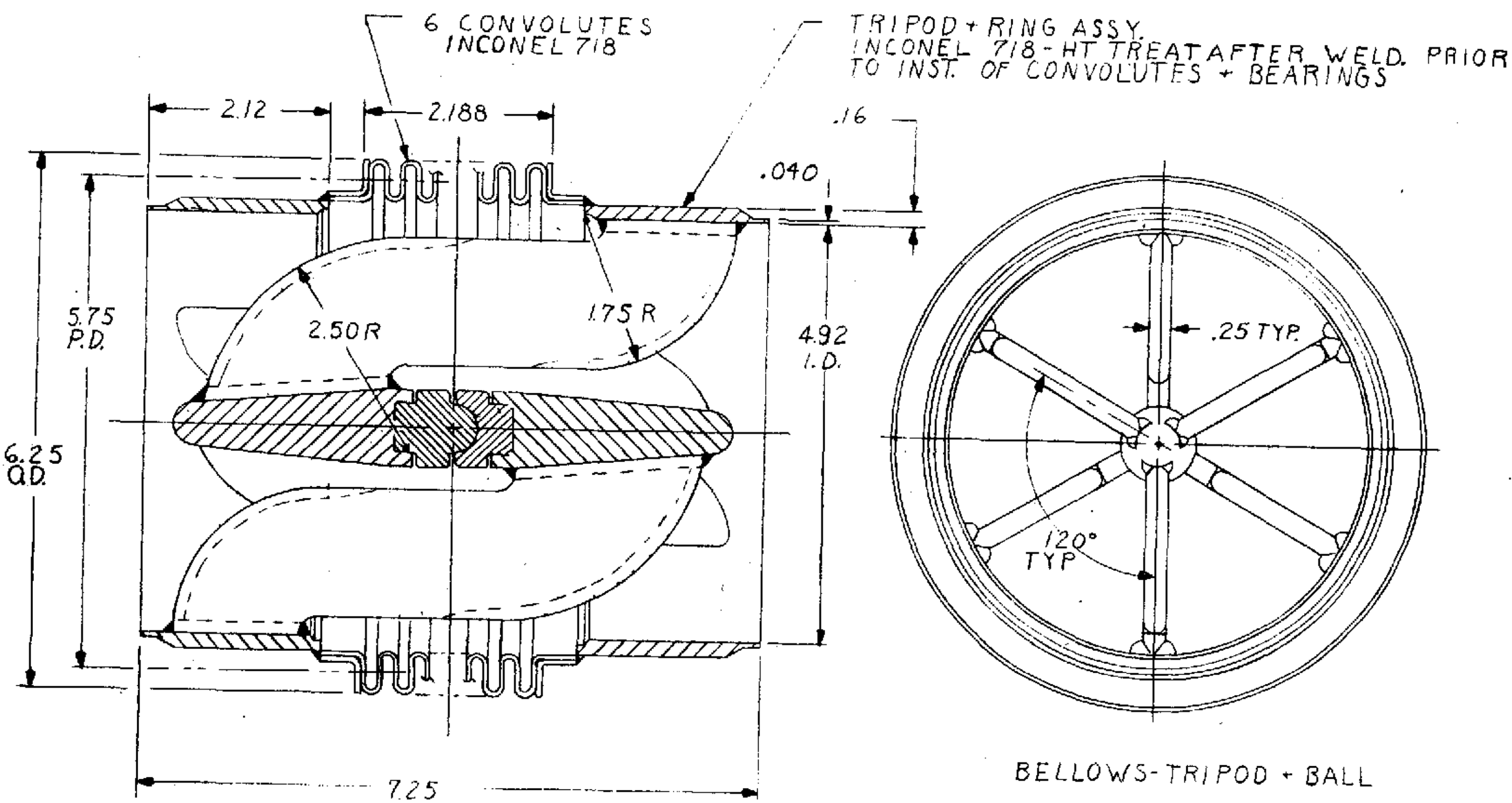

Fig. 1. Sketch of high pressure constrained bellows.

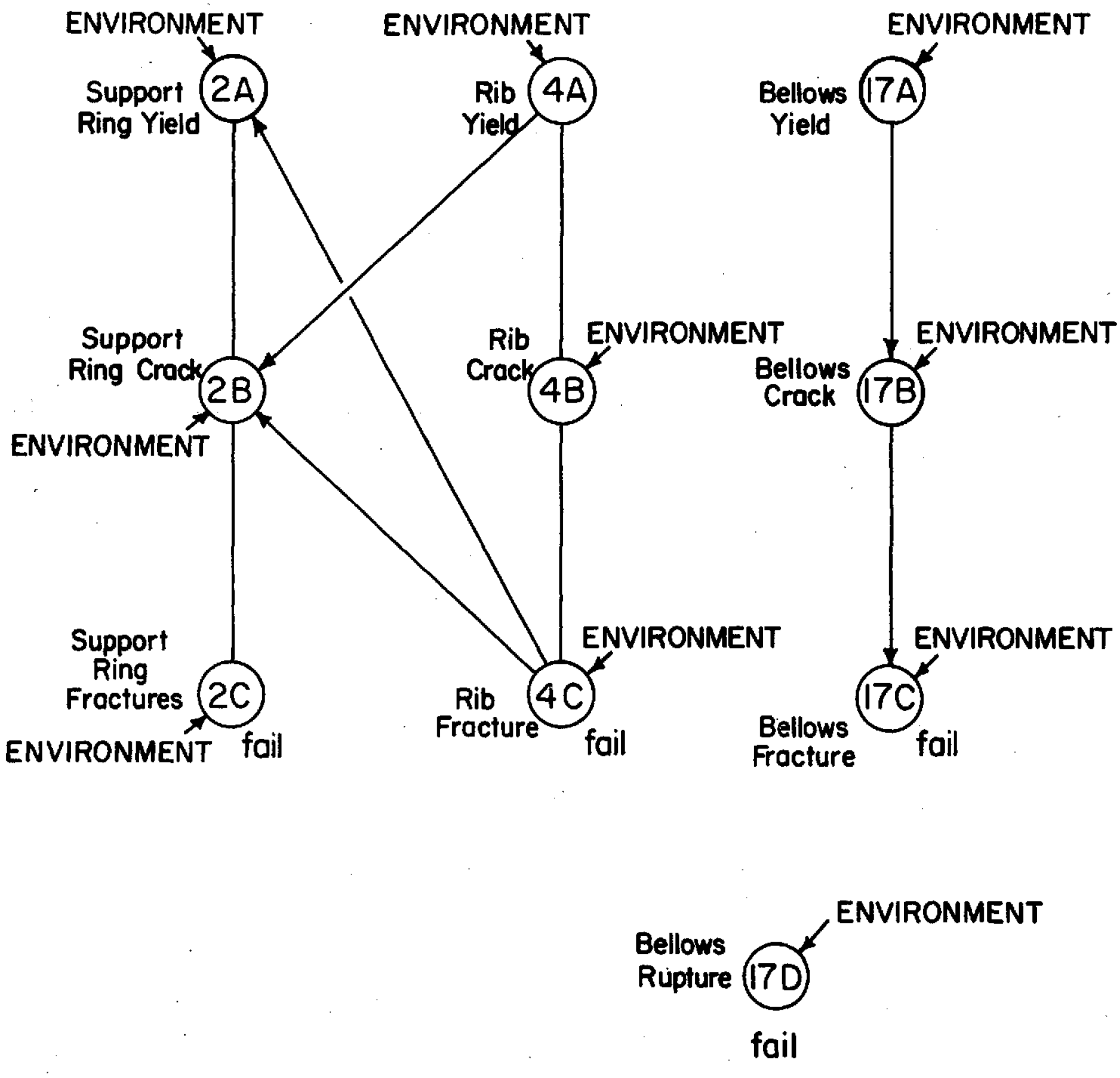

Fig. 2. Abbreviated version of constrained bellows graphical node network.

SMALL MATRIX (ORIGINAL)

CAUSES EFFECTS

		20	23	32	40	2A	2B	2C	4A	4B	4C	17A	17B	17C	17D	Succ.
Vibration-Ext.	20	0	0	0	0	0	1	0	0	1	0	0	1	0	0	1
Pressure Var.	23	0	0	0	0	1	1	0	1	1	0	1	1	0	1	0
Projectile-Int.	32	0	0	0	0	1	1	0	1	1	0	1	1	0	1	0
Starting Shock	40	0	0	0	0	0	1	0	1	1	0	1	1	0	0	0
Inlet Sup.Ring Yield	2A	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Inlet Sup. Ring Crack	2B	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
Inlet Sup. Ring Fract.	2C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rib Yield	4A	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0
Rib Crack	4B	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
Rib Fract.	4C	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
Bellows Yield	17A	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
Bellows Crack	17B	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Bellows Fract.	17C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Bellows Rupt.	17D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Success	S	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
								$\bar{1}$						$\bar{1}$	$\bar{2}$	

Figure 3

Small Interactive Matrix for Constrained Bellows.

"TOTAL" SMALL MATRIX

$$\text{each element} = \sum_{n=1}^{6} \text{corresponding element of (small matrix)}^{n}$$

$$\sum_{n=7}^{\infty} = 0$$

CAUSES		20	23	32	40	2A	2B	2C	4A	4B	4C	17A	17B	17C	17D	S
Vibration-Ext.	20	0	0	0	0	1	3	3	0	1	1	0	1	1	0	1
Pressure Var.	23	0	0	0	0	3	7	7	1	2	2	1	2	2	1	0
Projectile-Int.	32	0	0	0	0	3	7	7	1	2	2	1	2	2	1	0
Starting Shock	40	0	0	0	0	2	6	6	1	2	2	1	2	2	0	0
Inlet Sup. Ring Yield	2A	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
Inlet Sup. Ring Crack	2B	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
Inlet Sup. Ring Fract.	2C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rib Yield	4A	0	0	0	0	1	3	3	0	1	1	0	0	0	0	0
Rib Crack	4B	0	0	0	0	1	2	2	0	0	1	0	0	0	0	0
Rib Fract.	4C	0	0	0	0	1	2	2	0	0	0	0	0	0	0	0
Bellows Yield	17A	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
Bellows Crack	17B	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Bellows Fract.	17C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Bellows Rupt.	17D	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Success	S	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							--- 32							--- 9 2		

EFFECTS

Figure 4

Summation of product matrices for small matrix (Figure 3) of constrained bellows

Effect \ Cause	1. Load	2. Temp.	3. Temp. Grad.	4. Hardness	5. Yield Strength	6. Fatigue Limit	7. Residual Stress	8. Stress State	9. Crack Length	10. Rim Contour	11. Rim Thickness
1. Load, ρ	0	0	0	0	0	0	0	0	0	0	0
2. Temperature, T	0	0	0	0	0	0	0	0	0	0	0
3. Temperature Gradient, $\frac{dT}{dR}$	0	0	0	0	0	0	0	0	0	0	0
4. Hardness, H	0	1	0	0	0	0	0	1	0	0	0
5. Yield Strength, Y	0	1	0	0	0	0	0	1	0	0	0
6. Fatigue Limit, F	0	1	0	0	0	0	0	0	0	0	0
7. Residual Stress, σ_r	0	1	1	0	0	0	0	0	0	0	0
8. Stress State, σ	1	0	1	0	0	0	1	0	0	1	1
9. Crack Length, c	0	0	0	0	0	1	0	1	0	0	0
10. Rim Contour, G	0	0	0	1	1	0	0	1	0	0	0
11. Rim Thickness, R	0	0	0	1	1	0	0	1	0	0	0
12 Shelled Tread	0	0	0	0	0	0	0	0	1	0	0
13. Thermal Cracks	0	0	0	0	0	0	0	0	1	0	0

Figure 5

Interactive matrix for railroad wheel.

Appendix A[2]

ABSTRACT

This report describes the procedures for using a digital computer program for simulating GERT networks. New and advanced GERT concepts are introduced.

The simulation program can accommodate GERT networks which have logical operations associated with the input side of a node and branching operations associated with the output side of a node. Logical operations associated with a node are defined in terms of the number of realizations of activities incident to the node that must occur before the node can be realized. A similar quantity is required for realizing the node after its first realization. The branching operation associated with a node is either DETERMINISTIC (all branches are taken that emanate from the node) or PROBABILISTIC (a selection of one of the branches emanating from the node is taken when the node is realized).

Branches of a GERT network are described in terms of a probability that the branch is realized; a time to perform the activity represented by the branch; a count designation and an activity number. The time associated with a branch can be a random variable. The count designator identifies a count set for which a counter is indexed every time the branch is realized. The activity number identifies nodes that are affected by the realization of the branch. Through activity numbers, a network can be modified during the simulation of the network.

GERT networks having the above characteristics are simulated by a program labeled GERTS III. GERTS III is a fundamental package and as such provides the foundation for building advanced network simulation programs. In this report GERTS III has been extended in three directions. First, a queue node capability was added resulting in the GERTS IIIQ program. Then cost information was added to obtain GERTS IIIC. The third extension involved the

inclusion of resource requirements for each activity and limited resources to perform the project. The simulation package to study resource allocation has been labeled GERTS IIIR.

Examples of the use of GERTS III, GERTS IIIQ, GERTS IIIC, and GERTS IIIR are presented in the report. The GERTS III programs are written in FORTRAN IV. The program has been exercised on the IBM 360/65 system. GERT networks with up to 1,000 nodes can be analyzed.

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THE GERT SIMULATION PROGRAMS:

GERTS III; GERTS IIIQ; GERTS IIIC; and GERTS IIIR

INTRODUCTION

The GERT simulation program is a general purpose program for simulating networks. The program is written in FORTRAN IV. The input to the program is a description of the network in terms of its nodes and branches along with control information for setting up the simulation conditions. Applications of earlier GERT simulation programs [7,8,9] resulted in the need for new network concepts and additional capability. This need has been satisfied with the completion of the GERT Simulation Program III hereafter referred to as GERTS III.

The following list describes the features in GERTS III.

1. Branches that are characterized by:
   a. A probability of being included in the network;
   b. A time required to complete the activity represented by the branch. The time is specified by defining a parameter set number and a distribution type;
   c. A counter type to identify the branch as belonging to a particular group of branches; and
   d. An activity number.
2. Nodes that are characterized by:
   a. The number of releases required to realize the node for the first time;
   b. The number of releases required to realize the node after the first time;
   c. The removal of events that are scheduled to release the node;
   d. The method for scheduling the activities emanating from the node (DETERMINISTIC or PROBABILISTIC); and
   e. The statistical quantities to be estimated for the node.

3. Modification of the network based on the occurrence of end of activity events during the simulation of the network.

4. A method for tracing a set of simulation runs.

5. Automatic printout of the description of the network and the final results.

During the research leading to GERTS III, the following concepts were explored:

1) nodes that provided a storage or queue capability - a Q-node;
2) costs associated with the performance of activities; and
3) activities that required resources.

It was found that GERTS III could be modified to allow the simulation of networks that involved these concepts and implementation proceeded on a limited scale. It was felt that separate programs should be maintained for these new concepts but that each should contain the basic GERT simulation program, GERTS III. The results of the exploratory research are: 1) GERTS IIIQ, a GERT network simulation program that includes Q-nodes; 2) GERTS IIIC, a GERT network simulation program that collects cost statistics; and 3) GERTS IIIR, a GERT network simulation program that involves resource allocation decisions.

The main purpose of this report is to describe the procedure for using GERTS III, GERTS IIIQ, GERTS IIIC, and GERTS IIIR. Since many new concepts associated with GERT have been developed it is necessary to describe these before proceeding with examples illustrating the use of the new programs.

OVERALL PROGRAM OPERATION

The GERTS III program performs a simulation of a network by advancing time from event to event. In simulation parlance this is termed a next event simulation. The events associated with a simulation of a GERT network are:

(1) Start of the simulation; (2) End of an activity; and (3) Completion of a simulation run of the network. Since GERTS III is a FORTRAN IV program the operating procedure is the standard FORTRAN operating procedure. Many concepts of GERTS III were adopted from GASP IIA [10].

The start event causes all source nodes to be realized and schedules the activities emanating from the source nodes according to the output type of the source node. The output type for all nodes is either DETERMINISTIC or PROBABILISTIC. In the former case, all activities emanating from the node are scheduled and in the latter case, only one of the activities emanating from the node is scheduled. By scheduling an activity is meant that an event "end of activity" is caused to occur at some future point in time. The simulation proceeds from event to event until the conditions which indicate that the simulation of the network is completed are obtained. The above process is then repeated for a specified number of simulations of the network.

As part of the input data, the number of releases required to realize a node is specified. Each time an end of activity event occurs, the number of releases for the end node of that activity is decreased by one. When the number of releases remaining is zero, the node is realized. At this time the number of releases is set equal to the number of releases required to realize the node after the first time, and the activities emanating from the node are scheduled. Again, the number of activities scheduled depends on the output type for the node.

For each activity scheduled, an end of activity event is put in a file containing all events in chronological order. The end of activity events are removed from the event file one at a time and at each removal instant, a test is performed to determine if a node is realized. If a node is not realized, the next event is removed from the event file. If a node is realized, activities from that node are scheduled and the simulation is continued. The simulation ends when a prescribed number of sink nodes have been realized. As part of the input data, the number of source nodes, sink nodes and nodes on which statistics are collected as well as their node numbers and the number of nodes required to realize the network are defined.

The above process describes one simulation of a network. The program is written to allow multiple simulations to be performed. The number of simulation runs to be performed is part of the input data. The GERT simulation program automatically initializes the pertinent variables in order that consecutive simulations of the same network can be performed and, if desired, permits simulations of different networks to be performed consecutively.

GERTS III NETWORK CHARACTERISTICS

GERT networks consist of nodes and directed branches. First consider the characteristics that describes a node. The number of releases associated with a node specifies the number of times activities incident to the node must be realized before the node can be realized. When the number of releases is 1, the input side of the node can be thought of as an OR operation. If the number of releases equals the number of activities incident to the node, the node can be thought of an as AND operator. However, it is permissible to specify the number of releases to be less than or greater than the number of activities incident to the node. For example, the number of releases can be 2 whereas the number of activities incident to the node could be 3. This would represent the case where if 2 of the 3 activities were realized, the node is realized. Alternatively, the number of releases can be 2 and the number of activities incident to the node could be 1. This would represent the case where the activity must be realized twice before the node is realized.

Figure 1 illustrates the node symbolism for GERTS III.

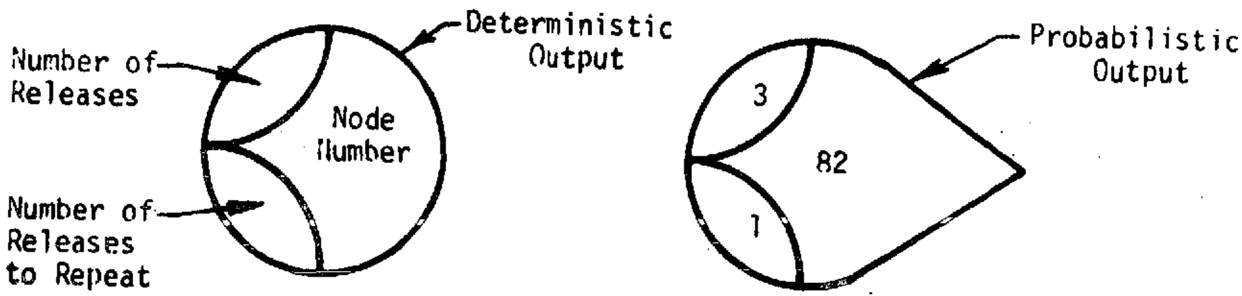


Figure 1. Node Symbolism for GERTS III

In Figure 1 it is seen that the semicircle, ↄ , on the output side of a node is used to represent a DETERMINISTIC output, and a lazy V, >, for a PROBABILISTIC output. Nodes are also characterized by their function in the network. A GERT analyst can specify a node as:

1. A Source Node;
2. A Sink Node;
3. A Statistics Node; or
4. A Mark Node.

Activities emanating from a source node are started at time zero. A sink node is a node that indicates that the network may be realized when it is realized. (NOTE: a sink node many have successor activities.) A statistics node is one on which statistics are maintained. All sink nodes are automatically made statistics nodes. A mark node establishes a reference time and permits the calculation of the time it takes to go between two nodes of the network.

For statistics nodes, GERTS III obtains statistical estimates associated with the time a node is realized. Five types of time statistics are possible:

F. The time of <u>first</u> realization of a node;

A. The time of <u>all</u> realizations of a node;

B. The time <u>between</u> realizations of a node;

I. The time <u>interval</u> required to go between two nodes in the network; and

D. The time <u>delay</u> from first activity completion on the node until the node is realized.

The nodes on which statistics are to be collected and the type of statistics desired are part of the description given to a node by the input to GERTS III. <u>They are not part of the graphical representation</u>.

The branches of GERT networks represent activities and/or information transfers. The term activity will be used to identify both. Activities emanate from a start node and are incident to an end node. Associated with activities are a probability that the activity will be realized given its start node is realized and a time to perform the activity given the activity is realized. For GERTS III the time variable is specified by a parameter set number and a distribution type. The following nine distribution types are available:

1. Constant;
2. Normal;
3. Uniform;
4. Erlang;
5. Lognormal;
6. Poisson;
7. Beta;
8. Gamma; and
9. Beta fitted to three parameters as in PERT.

The parameter set number along with the distribution type completely describe the time variable associated with an activity. Each distribution type specifies the arrangement of the parameters in a parameter set. With GERTS III, two additional characteristics can be associated with an activity.

These are a counter type and an activity number.

The counter type number specifies the counter to be increased by 1 every time the activity is realized. The number of counter types permitted is limited to 4*. Any number of activities may be associated with a counter type.** Statistics are automatically kept on the counter types. At the end of all simulation runs, the average and standard deviation of the number of times a counter type was realized prior to the realization of each node for which statistics are collected is determined and printed. In addition, the minimum and maximum number of times activities having the specified counter type were realized during a simulation is printed. Since the number of counts is always referenced to the realization of a node, the number of counts occurring prior to the realization of a node may be different in different simulation runs due to the sequence in which the nodes are realized.

Activity numbers are given to activities to permit network modifications based on the realization of the activity. Specification of an activity number does not automatically indicate that the network will be modified. However, only activities with activity numbers can cause the network to be modified. Network modification involves the replacing of a node by another node on the output side only.*** Thus when a node is realized, the activities to be started depend on the modifications that have taken place. For example if node 8 replaces node 5 then when node 5 is realized the activities emanating from node 8 are scheduled to start. A node may be changed many times before it is actually realized.

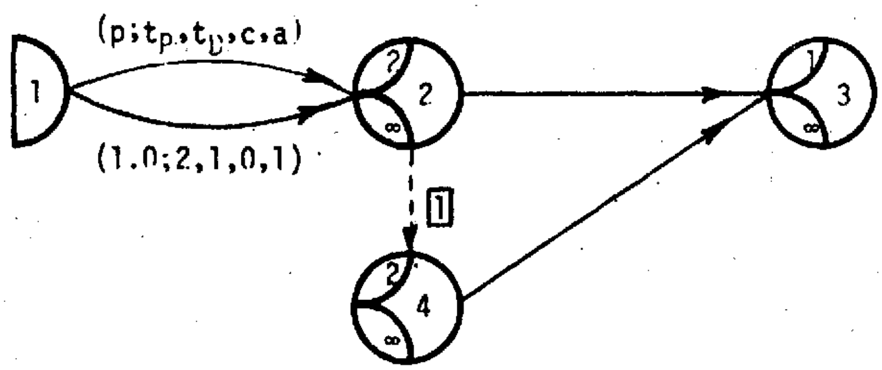
---

* Changes in the dimensions of two arrays can be made to increase this value.

** Activities incident to nodes on which delay statistics are collected or to Q-nodes may not have counter types associated with them.

*** The program can be modified to change the input side of a node also, [9, p. 57]. This involves decisions on the part of the user as to the number of releases remaining on the input side. Ref. [6] contains an example in which the input side of a node was modified.

The activity number causing the network modification along with all the nodes to be replaced, and the nodes to be inserted, are specified by the user. The method for incorporating network modifications is described later in the program operating procedure section. Figure 2 illustrates the branch and node modification notation that will be used throughout this report. Modifications will be shown by a dashed branch with the activity number attached in a square. The modification in Figure 2 is read "the output of node 2 is replaced by node 4 when activity 1 is realized".



LEGEND

p = probability of realization
t_p = parameter set for time
t_D = distribution type
c = counter type
a = activity number

Node A is replaced by node B when activity with activity number 1 is realized.

Figure 2. Illustration of Branch Descriptors and Network Modification Symbolism.

As an illustration of these new characteristics, consider the network of Figure 3.

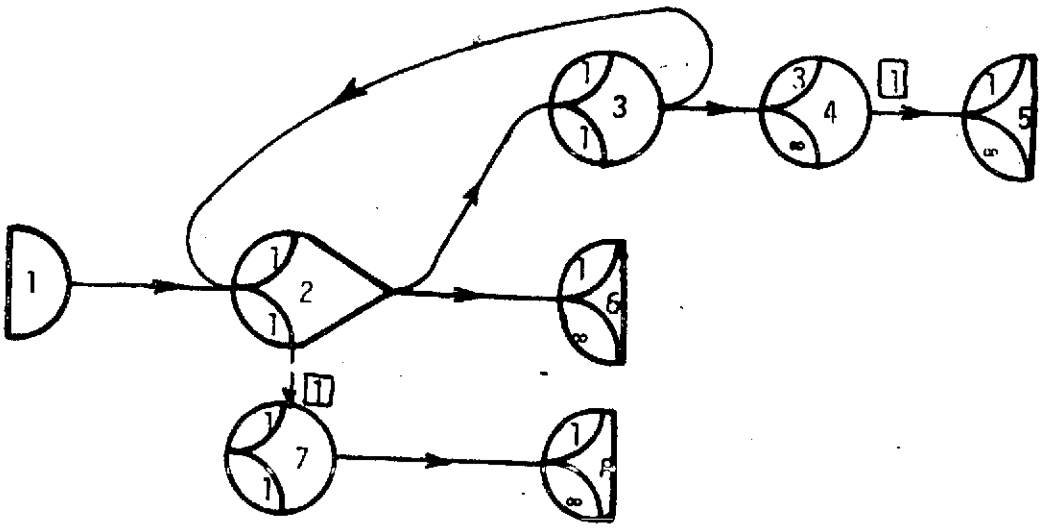


Figure 3. Network Containing Information Branches in Addition to Activities.

This network represents the changing of the network structure when the self-loop about node 2 is taken three times and is accomplished in the following manner. The output of node 3 is DETERMINISTIC so that every time node 3 is realized both branches emanating from node 3 are taken. The branch from node 3 to node 4 is used to count the number of times node 3 is realized. Node 4 is only realized when the branch incident to it is realized three times (of course, this corresponds to three traversals of the self-loop). When node 4 is realized, the activity labeled activity number 1 (a 1 on the network in this case) causes node 2 to be replaced by node 7, and the objective of changing the network is achieved.* Of significant importance in the above network is the incorporation on the network of branches representing activities and branches representing

* Care is required here to ensure that the branch from node 4 to node 5 is realized prior to the realization of the branch from node 3 to node 2. If a zero time is associated with both branches, normal operation would have the branch from node 3 to node 2 realized first since it was scheduled first. By assigning a small negative time (-.000001) to branch from node 4 to node 5, the desired ordering can be obtained.

information transfers. The inclusion of different types of branches within a GERT network expands the network modeling capability within the GERT framework.

Input to GERTS III and Limitations

The input requirements for GERTS III consist of at most 7 different types of data cards. These seven cards describe the network and the control information for performing the simulation. A general description of each card is provided below. In Appendix A, a complete description for each field of each Data Card type is presented.

Data Card Type	General Description
1	Identification Information, number of times simulation is to be performed and an initial random number seed (1 card).
2	General node, counter and network modification data (1 card).
3	Description of each node (1 card for each node).
4	Parameters of time variables associated with activities (1 card for each parameter set).
5	Description of each activity (1 card for each activity).
6	Network modifications desired (1 card for each activity that modifies network. If none, no Data Card Type 6 is required).
7	Run numbers to be traced (1 card only if tracing is requested by using a negative project number).

The dimensions of the GERTS III program have been set to allow for a maximum of 999 nodes, 999 activities, 4 counter types, collections of statistics on 100/(number of counter types + 1) nodes, and 300 parameter sets.

Examples of GERTS III

In previous reports examples were given that illustrated the use of the GERT Simulation Program to model to:

1. The modification of a project based on elapsed time [9];
2. The modification of a network based on the realization of the first of two activities [9];
3. The starting of a phase of a project based on progress to date [9];

4. Multiple modifications of a node during one realization of a network [9];
5. The Planning R & D Projects [3];
6. An advertising promotion in studying consumer brand choice [4]; and
7. A manufacturing process [11].

During the past year, the GERTS program has been used to analyze segments of a University [1], a product development problem for a large computer manufacturer, the R & D program for a weapons system and maintenance and checkout operations.

In this report, two examples are presented that demonstrate the new concepts included in GERTS III. The examples are:

1. Illustration of the features of GERTS III; and
2. Analysis and sequencing of space experiments.

Example 1. Illustration of the Features of GERTS III.

Figure 4 shows the network to be analyzed in Example 1. The source node for the network is node 2 and the sink node is node 12. From node 2 three activities emanate which are performed simultaneously. These activities cause nodes 3, 4, and 5 to be realized. The activities emanating from nodes 3, 4, and 5 are all incident to node 6. The number of releases required to realize node 6 is three, therefore node 6 will only be realized when all three activities incident to node 6 are realized. For this example, we desire to obtain statistics on the time delay between the completion of the first activity incident to node 6 and the time node 6 is realized. To obtain these statistics, node 6 is defined as a statistics node with delay statistics (code D) desired.

It is also desired to collect statistics on the time required to go from node 7 to node 11. To accomplish this node 7 is defined as a mark node (node type 4) and node 11 is defined as a statistics node with the statistics

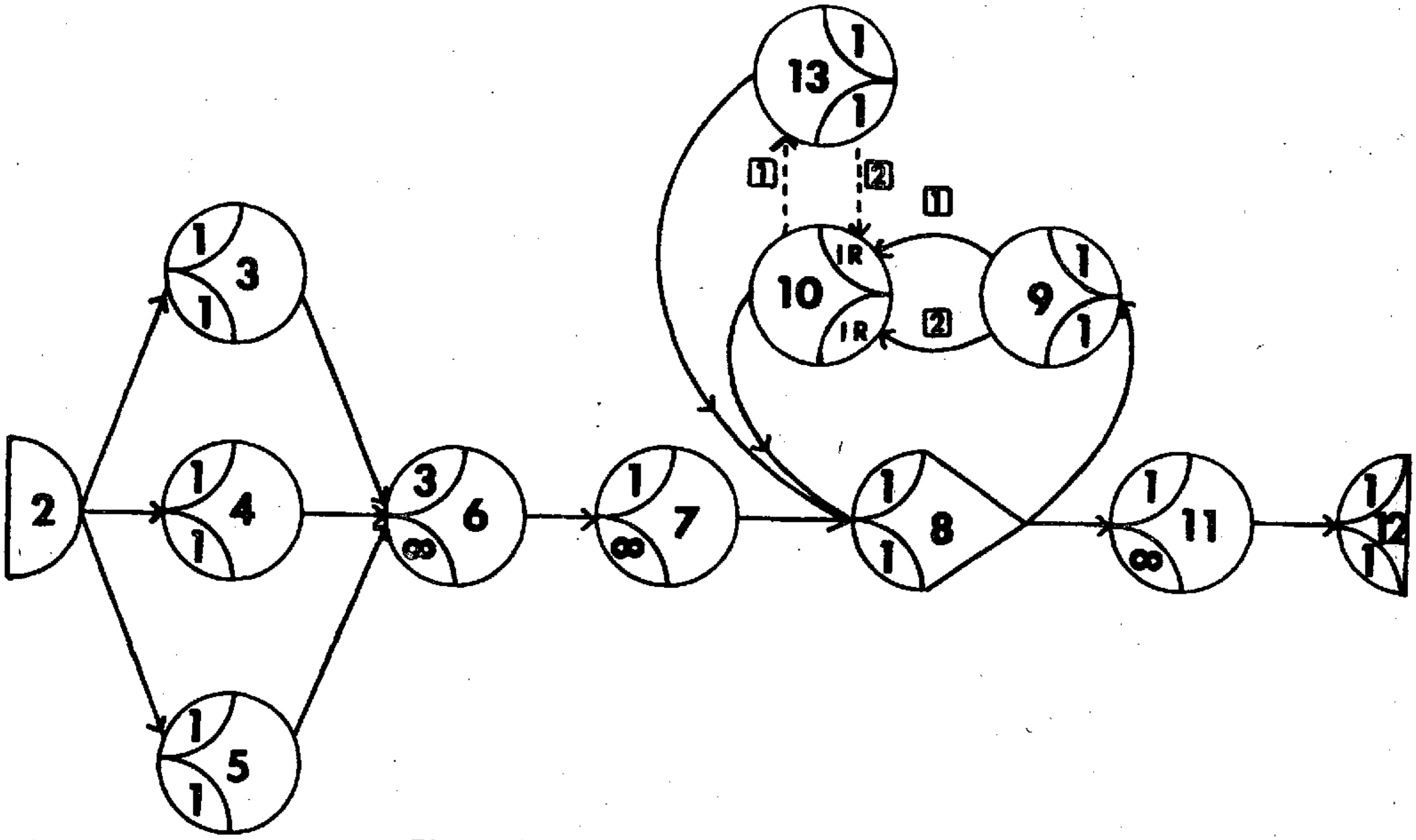


Figure 4. GERT Network for Example 1

calculated on an interval basis (code I). The statistical quantities collected at node 11 will be the interval of time from the realization of node 7 to the realization of node 11. If alternative paths existed between node 7 and 11, the interval node would collect statistics on the time required to traverse the separate paths. This will be further illustrated in Example 3.

Node 8 has a probabilistic output side so that either activity emanating from node 8 can be taken. For those situations where the feedback path is taken, it is desired to determine the time required to traverse the feedback path. This is accomplished by making node 8 a statistics node with statistics collected on time between realization of node 8 (code B). Nodes 9 and 10 were also defined as statistics nodes. For node 9 statistics are collected on its first realizations (code F) and for node 10 statistics are collected on all realizations (code A). Thus, this example includes all the types of statistical calculations that can be included within the GERTS III program. Since node 12 is a sink node, statistics will automatically be collected on it. For this example, statistics on the first realization of node 12 were specified. If the type of statistics desired is not specified by the input information, the GERTS III program assumes that statistics on first realization are desired, i.e., the default condition is first realization.

Also included in this example is the network modification feature and the stopping of an activity in progress. If the top activity (activity 1) from node 9 to node 10 is realized first then node 10 is replaced by node 13*. If the bottom activity (activity number 2) is realized first then node 10 remains in the network. If node 10 had been replaced by node 13 then

* For nodes on feedback paths, the input and output sides are reversed. However, the number of realizations to cause the node to be realized for the first time is still indicated by the top number.

node 10 is reinstated into the network. The network modification is implemented by assigning activity numbers to the branches between node 9 and node 10. When either of these activities are completed, the network modification is implemented and the other activity is stopped since node 10 has an "R" assigned to it. The removal of scheduled activities incident to a node applies to all realizations of the node.

A listing of the input cards for this example is shown in Figure 5. The description of the network that is printed by the GERTS III program is shown in Figures 6 and 7. Figure 8 presents a trace of a simulation run for this network. We will use this trace to describe the operating procedure of the GERTS III program in simulating the network shown in Figure 14.

The simulation begins by scheduling end of activity completion events from each source node. For Example 1, the source node is node 2 and end of activity events are scheduled for the activities from node 2 to node 3, node 2 to node 4, and node 2 to node 5. To obtain the time for each of these events, samples are drawn from: 1) a normal distribution using parameter set 1, 2) the Erlang distribution using parameter set 2, and 3) the uniform distribution using parameter set 3.

The trace of the simulation starts with the first end of activity event. This is seen to be the activity that is incident to node 5 and the event occurs at time 1.88. Since node 5 had its number of releases equal to 1, node 5 is realized and the activity from node 5 to node 6 can be initiated. An end of activity event for this activity is then scheduled by the program. At time 4.31, the activity on node 4 is completed as shown by the second line in the trace of Figure 8. At time 7.88 the activity from node 5 to node 6 was completed and we have the first activity incident to node 6 being completed. (To determine from the trace that this was the activity from node 5

```
ALL FEATURES S  -1 5201970 500  11  40      1267.         1    EX 1  10
 13  1  1  1  6  1  1                                     2    EX 1  20
  2  1     0                                                   EX 1  30
  3     1 10                                                   EX 1  40
  4     1 10                                                   EX 1  50
  5     1 10                                                   EX 1  60
  6  3  3  0    1     1 D                                      EX 1  70
  7  4  1 10                                                   EX 1  80
  8  3  1 1P    1     1 R                                 3    EX 1  90
  9  3  1 10   27     5 F                                      EX 1 100
 10  3  1 10R  30     2 A                                      EX 1 110
 11  3  1  0    8     2 I                                      EX 1 120
 12  2  1  0   35     2 A                                      EX 1 130
 13     1 10                                                   EX 1 140
  0                                                            EX 1 150
    10          0       100          1                         EX 1 160
     2          0       100         2                          EX 1 170
     3          0         5          5                         EX 1 180
     4          0       100         1                          EX 1 190
     5          0       100                                    EX 1 200
     6          0       100                               4    EX 1 210
 .6425       -5.0       5.0      1.492                         EX 1 220
     8          0       100          6                         EX 1 230
     3          2         5                                    EX 1 240
     0                                                         EX 1 250
 1.358        0.0     100.0      0.218                         EX 1 260
     1      2  3  1  2                                         EX 1 270
     1      2  4  2  4                                         EX 1 280
     1      2  5  3  3                                         EX 1 290
     1      3  6  4  4                                         EX 1 300
     1      4  6  5  6                                         EX 1 310
     1      5  6  6  1                                         EX 1 320
     1      6  7 11  5                                         EX 1 330
     1      7  8  8  1                                    5    EX 1 340
      6     8  9  9  9                                         EX 1 350
      4     9 11  7  8                                         EX 1 360
     1      9 10  3  7    1                                    EX 1 370
     1      9 10  9  1    2                                    EX 1 380
     1     10  8 10  1                                         EX 1 390
     1     11 12  1  2                                         EX 1 400
     1     13  8  1  1  1                                      EX 1 410
            0                                                  EX 1 420
 1 10 13  0                                                    EX 1 430
 2 13 10  0                                               6    EX 1 440
 0                                                             EX 1 450
 1  5                                                     7    EX 1 460
```

Figure 5. Input Data for Example 1

GERT SIMULATION PROJECT -1 BY ALL FEATURES
DATE 5/ 20/ 1970

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 13
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 6 NODES
NUMBER OF PARAMETER SETS IS 11
INITIAL RANDOM NUMBER IS 1267 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	D		
4	1	1	D		
5	1	1	D		
6	3	9999	D		D
7	-1	-1	D		
8	1	1	P		B
9	1	1	D		F
10	1	1	D	R	A
11	1	9999	D		I
12	1	9999	D		A
13	1	1	D		

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
12

STATISTICS COLLECTED ALSO ON NODES
6 11 10 9 8

Figure 6. Echo Check for Example 1

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	10.0000	0.0	100.0000	0.1000
2	2.0000	0.0	100.0000	2.0000
3	3.0000	0.0	5.0000	0.5000
4	4.0000	0.0	100.0000	1.0000
5	5.0000	0.0	100.0000	0.0
6	6.0000	0.0	100.0000	0.0
7	0.6425	-5.0000	5.0000	1.4920
8	8.0000	0.0	100.0000	0.6000
9	3.0000	2.0000	5.0000	0.0
10	0.0	0.0	0.0	0.0
11	1.3580	0.0	100.0000	0.2180

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	2	0	0	1.0000
2	4	2	4	0	0	1.0000
2	5	3	3	0	0	1.0000
3	6	4	4	-1000	0	1.0000
4	6	5	6	-1000	0	1.0000
5	6	6	1	-1000	0	1.0000
6	7	11	5	0	0	1.0000
7	8	8	1	0	0	1.0000
8	9	9	9	0	0	0.6000
8	11	7	8	0	0	0.4000
9	10	3	7	0	1	1.0000
9	10	9	3	0	2	1.0000
10	8	10	1	0	0	1.0000
11	12	1	2	0	0	1.0000
13	8	1	1	1	0	1.0000

NETWORK MODIFICATIONS

ACTIVITY	NODE	FILE	NODE	FILE	NODE	FILE	NODE	FILE	NODE	FILE	NODE	FILE	NODE	FILE	NODE	FILE	NODE
1	10	13															
2	13	10															

Figure 7. Further Echo Check for Example 1

```
AT TIME      4.31 ACTIVITY ON NODE    4 WITH ATTRIBUTES    2     4     0     0 WAS REALIZED ON RUN    1
AT TIME      7.88 ACTIVITY ON NODE    6 WITH ATTRIBUTES    6     1-1000      0 WAS REALIZED ON RUN    1
AT TIME      8.31 ACTIVITY ON NODE    6 WITH ATTRIBUTES    5     6-1000      0 WAS REALIZED ON RUN    1
AT TIME     10.16 ACTIVITY ON NODE    3 WITH ATTRIBUTES    1     2     0     0 WAS REALIZED ON RUN    1
AT TIME     12.09 ACTIVITY ON NODE    6 WITH ATTRIBUTES    4     4-1000      0 WAS REALIZED ON RUN    1
AT TIME     13.73 ACTIVITY ON NODE    7 WITH ATTRIBUTES   11     5     0     0 WAS REALIZED ON RUN
AT TIME     21.73 ACTIVITY ON NODE    8 WITH ATTRIBUTES    8     1     0     0 WAS REALIZED ON RUN
AT TIME     25.56 ACTIVITY ON NODE    9 WITH ATTRIBUTES    9     9     0     0 WAS REALIZED ON RUN    1
AT TIME     28.79 ACTIVITY ON NODE   10 WITH ATTRIBUTES    3     7     0     1 WAS REALIZED ON RUN    1
AT TIME     38.79 ACTIVITY ON NODE    8 WITH ATTRIBUTES    1     1     1     0 WAS REALIZED ON RUN    1
AT TIME     41.72 ACTIVITY ON NODE    9 WITH ATTRIBUTES    9     9     0     0 WAS REALIZED ON RUN    1
AT TIME     43.98 ACTIVITY ON NODE   10 WITH ATTRIBUTES    3     7     0     1 WAS REALIZED ON RUN    1
AT TIME     53.98 ACTIVITY ON NODE    8 WITH ATTRIBUTES    1     1     1     0 WAS REALIZED ON RUN    1
AT TIME     57.10 ACTIVITY ON NODE    9 WITH ATTRIBUTES    9     9     0     0 WAS REALIZED ON RUN    1
AT TIME     59.54 ACTIVITY ON NODE   10 WITH ATTRIBUTES    9     3     0     2 WAS REALIZED ON RUN    1
AT TIME     59.54 ACTIVITY ON NODE    8 WITH ATTRIBUTES   10     1     0     0 WAS REALIZED ON RUN    1
AT TIME     62.19 ACTIVITY ON NODE    9 WITH ATTRIBUTES    9     9     0     0 WAS REALIZED ON RUN    1
AT TIME     65.39 ACTIVITY ON NODE   10 WITH ATTRIBUTES    3     7     0     1 WAS REALIZED ON RUN    1
AT TIME     75.39 ACTIVITY ON NODE    8 WITH ATTRIBUTES    1     1     1     0 WAS REALIZED ON RUN    1
AT TIME     78.23 ACTIVITY ON NODE    9 WITH ATTRIBUTES    9     9     0     0 WAS REALIZED ON RUN    1
AT TIME     81.89 ACTIVITY ON NODE   10 WITH ATTRIBUTES    3     7     0     1 WAS REALIZED ON RUN    1
AT TIME     91.89 ACTIVITY ON NODE    8 WITH ATTRIBUTES    1     1     1     0 WAS REALIZED ON RUN    1
AT TIME     95.05 ACTIVITY ON NODE    9 WITH ATTRIBUTES    9     9     0     0 WAS REALIZED ON RUN    1
AT TIME     97.56 ACTIVITY ON NODE   10 WITH ATTRIBUTES    3     7     0     1 WAS REALIZED ON RUN    1
AT TIME    107.56 ACTIVITY ON NODE    8 WITH ATTRIBUTES    1     1     1     0 WAS REALIZED ON RUN    1
AT TIME    112.56 ACTIVITY ON NODE   11 WITH ATTRIBUTES    7     8     0     0 WAS REALIZED ON RUN    1
AT TIME    122.65 ACTIVITY ON NODE   12 WITH ATTRIBUTES    1     2     0     0 WAS REALIZED ON RUN    1
AT TIME      1.29 ACTIVITY ON NODE    4 WITH ATTRIBUTES    2     4     0     0 WAS REALIZED ON RUN    2
AT TIME      2.98 ACTIVITY ON NODE    5 WITH ATTRIBUTES    3     3     0     0 WAS REALIZED ON RUN    2
AT TIME      4.29 ACTIVITY ON NODE    6 WITH ATTRIBUTES    5     6-1000      0 WAS REALIZED ON RUN    2
AT TIME      8.98 ACTIVITY ON NODE    6 WITH ATTRIBUTES    6     1-1000      0 WAS REALIZED ON RUN    2
AT TIME     10.11 ACTIVITY ON NODE    3 WITH ATTRIBUTES    1     2     0     0 WAS REALIZED ON RUN    2
AT TIME     20.39 ACTIVITY ON NODE    6 WITH ATTRIBUTES    4     4-1000      0 WAS REALIZED ON RUN    2
AT TIME     21.97 ACTIVITY ON NODE    7 WITH ATTRIBUTES   11     5     0     0 WAS REALIZED ON RUN    2
AT TIME     29.97 ACTIVITY ON NODE    8 WITH ATTRIBUTES    8     1     0     0 WAS REALIZED ON RUN    2
AT TIME     32.77 ACTIVITY ON NODE    9 WITH ATTRIBUTES    9     9     0     0 WAS REALIZED ON RUN    2
AT TIME     35.39 ACTIVITY ON NODE   10 WITH ATTRIBUTES    9     3     0     2 WAS REALIZED ON RUN    2
AT TIME     35.39 ACTIVITY ON NODE    8 WITH ATTRIBUTES   10     1     0     0 WAS REALIZED ON RUN    2
AT TIME     38.47 ACTIVITY ON NODE   11 WITH ATTRIBUTES    7     8     0     0 WAS REALIZED ON RUN
AT TIME     48.63 ACTIVITY ON NODE   12 WITH ATTRIBUTES    1     2     0     0 WAS REALIZED ON RUN
AT TIME      3.57 ACTIVITY ON NODE    5 WITH ATTRIBUTES    3     3     0     0 WAS REALIZED ON RUN    3
AT TIME      9.57 ACTIVITY ON NODE    6 WITH ATTRIBUTES    6     1-1000      0 WAS REALIZED ON RUN    3
AT TIME      9.88 ACTIVITY ON NODE    3 WITH ATTRIBUTES    1     2     0     0 WAS REALIZED ON RUN    3
AT TIME     14.02 ACTIVITY ON NODE    4 WITH ATTRIBUTES    2     4     0     0 WAS REALIZED ON RUN    3
AT TIME     15.08 ACTIVITY ON NODE    6 WITH ATTRIBUTES    4     4-1000      0 WAS REALIZED ON RUN    3
AT TIME     20.02 ACTIVITY ON NODE    6 WITH ATTRIBUTES    5     6-1000      0 WAS REALIZED ON RUN    3
AT TIME     21.35 ACTIVITY ON NODE    7 WITH ATTRIBUTES   11     5     0     0 WAS REALIZED ON RUN    3
AT TIME     29.35 ACTIVITY ON NODE    8 WITH ATTRIBUTES    8     1     0     0 WAS REALIZED ON RUN    3
AT TIME     31.92 ACTIVITY ON NODE    9 WITH ATTRIBUTES    9     9     0     0 WAS REALIZED ON RUN    3
AT TIME     35.10 ACTIVITY ON NODE   10 WITH ATTRIBUTES    9     3     0     2 WAS REALIZED ON RUN    3
AT TIME     35.10 ACTIVITY ON NODE    8 WITH ATTRIBUTES   10     1     0     0 WAS REALIZED ON RUN    3
AT TIME     39.08 ACTIVITY ON NODE    9 WITH ATTRIBUTES    9     9     0     0 WAS REALIZED ON RUN    3
AT TIME     42.70 ACTIVITY ON NODE   10 WITH ATTRIBUTES    3     7     0     1 WAS REALIZED ON RUN    3
AT TIME     52.70 ACTIVITY ON NODE    8 WITH ATTRIBUTES    1     1     1     0 WAS REALIZED ON RUN    3
AT TIME     53.36 ACTIVITY ON NODE   11 WITH ATTRIBUTES    7     8     0     0 WAS REALIZED ON RUN    3
AT TIME     63.29 ACTIVITY ON NODE   12 WITH ATTRIBUTES    1     2     0     0 WAS REALIZED ON RUN    3
AT TIME      4.99 ACTIVITY ON NODE    5 WITH ATTRIBUTES    3     3     0     0 WAS REALIZED ON RUN    4
AT TIME      5.52 ACTIVITY ON NODE    4 WITH ATTRIBUTES    2     4     0     0 WAS REALIZED ON RUN    4
AT TIME      7.52 ACTIVITY ON NODE    6 WITH ATTRIBUTES    5     6-1000      0 WAS REALIZED ON RUN    4
AT TIME     10.10 ACTIVITY ON NODE    3 WITH ATTRIBUTES    1     2     0     0 WAS REALIZED ON RUN    4
AT TIME     10.99 ACTIVITY ON NODE    6 WITH ATTRIBUTES    6     1-1000      0 WAS REALIZED ON RUN    4
AT TIME     13.71 ACTIVITY ON NODE    6 WITH ATTRIBUTES    4     4-1000      0 WAS REALIZED ON RUN    4
AT TIME     15.03 ACTIVITY ON NODE    7 WITH ATTRIBUTES   11     5     0     0 WAS REALIZED ON RUN    4
AT TIME     23.03 ACTIVITY ON NODE    8 WITH ATTRIBUTES    8     1     0     0 WAS REALIZED ON RUN    4
AT TIME     24.43 ACTIVITY ON NODE   11 WITH ATTRIBUTES    7     8     0     0 WAS REALIZED ON RUN    4
AT TIME     34.35 ACTIVITY ON NODE   12 WITH ATTRIBUTES    1     2     0     0 WAS REALIZED ON RUN    4
AT TIME      1.04 ACTIVITY ON NODE    5 WITH ATTRIBUTES    3     3     0     0 WAS REALIZED ON RUN    5
AT TIME      2.91 ACTIVITY ON NODE    4 WITH ATTRIBUTES    2     4     0     0 WAS REALIZED ON RUN    5
AT TIME      3.91 ACTIVITY ON NODE    6 WITH ATTRIBUTES    5     6-1000      0 WAS REALIZED ON RUN    5
AT TIME      7.04 ACTIVITY ON NODE    6 WITH ATTRIBUTES    6     1-1000      0 WAS REALIZED ON RUN    5
AT TIME     10.13 ACTIVITY ON NODE    3 WITH ATTRIBUTES    1     2     0     0 WAS REALIZED ON RUN    5
AT TIME     18.45 ACTIVITY ON NODE    6 WITH ATTRIBUTES    4     4-1000      0 WAS REALIZED ON RUN
AT TIME     19.76 ACTIVITY ON NODE    7 WITH ATTRIBUTES   11     5     0     0 WAS REALIZED ON RUN
AT TIME     27.76 ACTIVITY ON NODE    8 WITH ATTRIBUTES    8     1     0     0 WAS REALIZED ON RUN    5
AT TIME     28.33 ACTIVITY ON NODE   11 WITH ATTRIBUTES    7     8     0     0 WAS REALIZED ON RUN    5
AT TIME     38.26 ACTIVITY ON NODE   12 WITH ATTRIBUTES    1     2     0     0 WAS REALIZED ON RUN    5
```

Figure 8. Tracing of Activity Completions for the Simulation of the Network in Example 1

to node 6, the attributes of the activity are examined). Since node 6 is a delay node, the program records the time 7.88 as the time of first completion of an activity that is incident to node 6. At time 8.31, the activity from node 4 to node 6 is completed. At time 10.16 the activity incident to node 3 is completed and node 3 is realized. The activity from node 3 to node 6 is then scheduled and is completed at time 12.09. This is the third activity that is realized incident to node 6 hence node 6 is realized. The time from the first activity completion on node 6 to the time that node 6 is realized is the delay time. This value is 4.21 (12.09 - 7.88) and is one sample of the delay time associated with node 6. Next the activity from node 6 to node 7 is scheduled. This activity is completed at time 13.78.

Node 7 is a mark node and the time 13.73 is identified with the path of activities following node 7. The activity emanating from node 7 is then completed at time 21.73 and node 8 is realized. This value is recorded for node 8 as the first time node 8 is realized since the time between realizations of node 8 is desired. From the trace, we see that node 9 is realized next at time 25.56. This indicates that the branching operation took the branch from node 8 to node 9 for this simulation of the network. Statistics are collected on node 9 which was realized at time 25.56. The two activities emanating from node 9 are then scheduled. Activity 1, the upper branch is completed first at time 28.79. This causes node 10 to be realized and the value of 28.79 is recorded as a time of realization of node 10. Since node 10 removes all activities scheduled to be completed that are incident to node 10, activity 2 is halted. Since activity 1 has been completed, node 10 is replaced by node 13 according to the prescribed network modification.

Branching from node 13 is now done. This is indicated by the trace by the attributes associated with the end of activity event on node 8 being those from node 13 to node 8. The time between realization of node 8 is collected and the current time used as the last time node 8 was realized. Again the branching process selects the activity from node 8 to node 9 and the loop around node 8 is traversed again. On this second traversal of the loop activity 1 again was completed before activity 2, and the branch from node 13 to node 8 is included in the netowrk. On the third traversal of the loop, activity 2 was completed before activity 1 and the branch from node 10 to node 8 which involved no time delay is included in the network. Finally at time 107.56, node 8 is realized and the branching process directs that the activity from node 8 to node 11 be completed. Node 11 is realized at time 112.56. Since node 11 is an interval node, a value is calculated which represents the time to go from node 7 to node 11. In this case it is 98.78 (112.56 - 13.78). The activity from node 11 to node 12 is scheduled and completed at 122.65. At this time node 12 is realized. Since node 12 is the sink node of the network and since it only takes one realization of the sink node to realize the network, the network is realized. The value of 122.65 is then recorded as 1 sample of the time to realize node 12 or equivalently the time to realize the network. This completes one simulation run of the network.

Several comments on the statistics collected on nodes 8, 9, and 10 are in order. For run 1 node 8 was realized seven times, therefore, six values were calculated for the time between realizations of node 8. For node 9, statistics are collected on the time of first realization therefore only the value 25.56 is recorded as the appropriate sample on run 1 for node 9. For node 10, all realization times are collected since node 10 is an ALL node. Thus the values 28.79 , 43.98 , 59.54 , 65.39 , 81.89 , and

97.56 are sample values regarding the realization of node 10.

In this example, all 9 distribution types were utilized to obtain samples for the time required to perform an activity. In Figure 8, a trace of 4 additional simulation runs is presented to indicate both the variability of the time required to perform an activity and the variability involved in the network structure due to the branching process and the network modification procedures.

The final GERTS summary report for Example 1 is presented in Figure 9 for 500 simulations of the network. The statistics presented for node 12 represents the values associated with the completion time of the network. From Figure 9, it is seen that node 12 has a probability of one of being realized as expected. The average time to realize node 12 was approximately 53.55 time units with a standard deviation of approximately 23.43 time units. In one simulation the network was realized in less than 30 time units and in another simulation it required over 148 time units to realize the network. Since node 12 is only realized once in each simulation there is no difference between statistics based on first realization and all realizations. In this simulation the branch from node 13 to node 8 was designated with a counter type 1. Statistics are automatically collected on the number of times that branch was taken prior to the realization of the node on which the statistics are collected. For node 12 it is seen that the average number of times the branch from node 13 to node 8 was taken prior to the realization of the network was .894. In some cases, the branch was never taken and in at least one simulation the branch was taken 7 times before the network was realized.

GERT SIMULATION PROJECT -1 BY ALL FEATURES
DATE 5/ 20/ 1970

FINAL RESULTS FOR 500 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
12	1.0000	53.5455	23.4290	500.	29.5463	148.3445	A
12	1	0.8940	1.2923	500.	0.0	7.0000	
6	1.0000	14.2856	3.8987	500.	4.2100	35.3449	D
6	1	0.0	0.0	500.	0.0	0.0	
11	1.0000	27.8881	23.1954	500.	8.0286	125.6182	I
11	1	0.8940	1.2923	500.	0.0	7.0000	
10	0.5660	46.5107	21.7802	697.	23.3519	132.5558	A
10	1	0.8838	1.2373	697.	0.0	6.0000	
9	0.5660	26.7693	3.7985	283.	21.7731	46.9250	F
9	1	0.0	0.0	283.	0.0	0.0	
8	1.0000	17.0576	7.1713	1197.	4.1900	44.8769	B
8	1	0.8881	1.2601	1197.	0.0	7.0000	

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
12	35.00	2.00	90	46	41	29	14	14	17	22	14	36	12
			9	14	7	8	12	4	12	11	9	3	5
			5	5	6	4	4	5	2	4	4	32	
6	1.00	1.00	0	0	0	0	1	0	0	0	0	5	84
			70	74	64	35	27	35	30	27	8	12	5
			6	2	5	2	2	0	3	0	0	3	
11	8.00	2.00	0	97	52	73	7	12	13	2	15	22	25
			32	8	11	6	5	3	13	9	13	6	5
			1	2	7	4	2	5	10	3	3	34	
10	30.00	2.00	183	45	52	23	20	10	29	41	27	27	27
			10	15	12	14	23	10	7	11	7	4	8
			7	11	7	6	4	2	3	4	5	43	
9	27.00	0.50	181	12	6	8	10	6	9	8	16	4	3
			2	5	1	0	2	2	1	0	0	0	0
			1	2	0	0	0	0	1	1	0	2	
8	1.00	1.00	0	0	0	0	24	124	82	20	0	0	0
			0	0	4	63	174	159	45	2	56	80	73
			71	41	31	33	29	33	12	12	6	23	

Figure 9. GERTS III Summary Report for Example 1

Statistics on node 6 show that the time between the first completion of an activity on node 6 and the time node 6 is realized required almost 7 time units. For the count statistics listed under node 6, it is seen that the branch from node 13 to node 8 was never taken prior to the realization of node 6. This is as expected since that branch follows node 6. Other items of interest from the final GERTS summary report will now be described. The probability associated with nodes 9 and 10 represent the probability that either of these nodes were realized in any simulation run. It is seen that branching around the loop from node 8 occurred in 56.6 percent of the runs. Even though statistics for node 10 are collected for all realizations, the probability of realizing node 10 on a simulation run is the probability of every realizing node 10, in that simulation run. If it is desired to obtain the average number of times node 10 was realized, this can be calculated from the number of observations divided by the number of simulation runs (697 divided by 500 for this example). The average time of realizing node 10 in a simulation run is the sum of all realization times of node 10 divided by the number of times node 10 is realized. This statistic is not an ordinary one for network models since it combines the time of first realization, second realization, and so on. Care must be taken when using these values.

Histograms for each of the statistics nodes are also presented in Figure 9. Consider the histogram for node 12 where the lower limit of the second cell is 35 and the cell width of each cell is 2. From the data presented, is seen that in 90 of the 500 simulation runs the realization time for node 12 and hence the network was less than 35 time units. In 46 other simulation runs the time to complete the network was between 35 and 37 time units. Other values can be read directly from Figure 9. This example demonstrates that a great deal of data can be obtained from GERTS III.

Example 2. Analysis and Sequencing of Space Experiments*

The performance of experiments in space by a spacecraft crew are almost always severely constrained by time. Many experiments are usually proposed by the scientific community and of those proposed a subset must be chosen for a given space mission. The sequencing of these experiments can be an important factor in determining the number of experiments that can be completed.

A GERT network of the sequence of experiments will be developed that permits the assessment of the time required to perform the experiments. In addition, information regarding the number of experiments that can be completed in a specified period of time will be determined. By modifying the sequence of experiments (which involves modifying the GERT network) an analysis can be performed on proposals for different sequencing procedures.

It will be assumed that there are three possible outcomes from the performance of an experiment: 1) successful completion; 2) failure; and 3) inconclusive results. If an experiment is successfully completed the next experiment in the sequence is performed. If a failure occurs, the experiment is scrubbed and the next experiment is then performed. If the results of an experiment are inconclusive, the experiment is repeated n times or until a success or failure occurs. The experiment is scrubbed if it is tried n times and the results are still inconclusive.

The GERT network for a three experiment program is shown in Figure 10. Node 2 is the start node and initiates a transfer to node 3. Node 3 represents the decision point for the first experiment. If the first experiment is successful, the activity from node 3 to node 4 is traversed. If the first experiment fails, then the activity from node 3 to node 19 is taken. The

* This example was developed by Mr. J. Ignizio in a seminar on GERT based on Mr. Ignizio's experience [5].

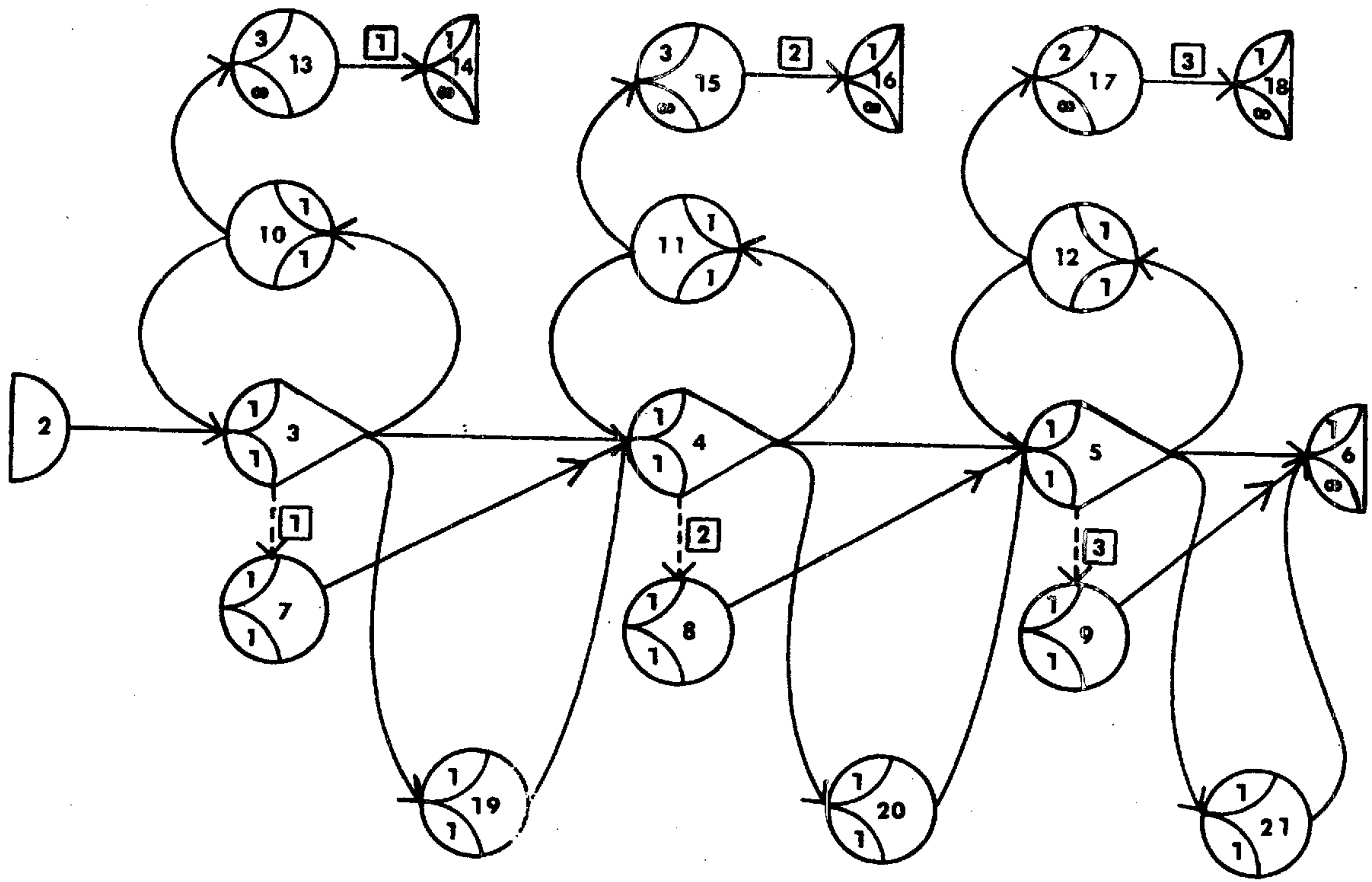


Figure 10. GERT Network for the Analysis and Sequencing of Space Experiments

second experiment is started by transferring from node 19 to node 4. If the results of the first experiment are inconclusive, the activity from node 3 to node 10 is taken. The output of node 10 is DETERMINISTIC; and both the first experiment is performed again and a signal to node 13 is sent to indicate that the first experiment has been performed once. Thus, for each experiment we will either transfer to node 4 or reach node 13. When node 13 is realized three times, the activity from node 13 to node 14 is traversed. This activity is labeled as activity 1 and causes the network to be modified by replacing node 3 with node 7. After this occurs when node 3 is realized, node 7 is in the network and a transfer to node 4 is caused. A similar discussion holds for experiments 2 and 3. From the network it is seen that nodes 19, 20, and 21 represent the failure of experiments 1, 2, and 3, respectively. Nodes 14, 16 and 18 represent the outcomes that inconclusive results were obtained after the maximum number of experiments could be performed for experiments 1, 2, and 3, respectively.

Table 1 presents the data for the experimental characteristics to be analyzed. In Figure 11, the input for Example 2 is presented. The GERTS III echo check for the description of the network is presented in Figures 12 and 13. Figure 14 presents a summary report describing the results from the GERTS III simulation of the sequence proposed for the space experiment program. From the output it is seen that experiment 1 failed 15.75% of the time and had inconclusive results 3.5% of the time. Therefore, it was successful 81% of the time. The time to complete experiment 1 is the time to reach node 4 of the network. Figure 14 shows that on the average it took over 46 time units to reach node 4 with a standard deviation of over 18 time units. In some instances it took as little as 24.5 time units and in others it took over 114 time units to complete the experiment. The number of times that experiment 1 was completed within given time intervals is presented in

```
SPACE EXPS        2 5201970 600    4  40      1267      1      EX 2  10
21   1   1   1 10          1                            2      EX 2  20
  2  1   0   0                                                 EX 2  30
  3  3   1   1P     5        1  A                              EX 2  40
  4  2   1   1P    25        3  A                              EX 2  50
  5  3   1   1P    37        3  A                              EX 2  60
  6  2   1   0     37        3  A                              EX 2  70
  7      1   10                                                EX 2  80
  8      1   10                                                EX 2  90
  9      1   10                                                EX 2 100
 10      1   10                                                EX 2 110
 11      1   10                                         3      EX 2 120
 12      1   10                                                EX 2 130
 13      3   0                                                 EX 2 140
 14  3   1   0      1        1  F                              EX 2 150
 15      3   0                                                 EX 2 160
 16  3   1   0     65        1  F                              EX 2 170
 17      2   0                                                 EX 2 180
 18  3   1   0     50        2  F                              EX 2 190
 19  3   1   10     1        1  A                              EX 2 200
 20  3   1   10    27        2  A                              EX 2 210
 21  3   1   10    41        3  A                              EX 2 220
  0                                                            EX 2 230
    10          5          20         2                        EX 2 240
    20         15          25         1                 4      EX 2 250
    15         10          30         3                        EX 2 260
     0                                                         EX 2 270
   1       2   3   1   2                                       EX 2 280
     6     3   6   2   2                                       EX 2 290
     3     3  10   4   1                                       EX 2 300
     1     3  19   4   1                                       EX 2 310
     5     4   5   3   2                                       EX 2 320
     4     4  11   4   1                                       EX 2 330
     1     4  20   4   1                                       EX 2 340
     7     5   6   4   1                                       EX 2 350
     2     5  12   4   1                                       EX 2 360
     1     5  21   4   1                                       EX 2 370
   1       7   4   2   2                                       EX 2 380
   1       8   5   3   2                                       EX 2 390
   1       9   6   4   1                                       EX 2 400
   1      10   3   1   2                                5      EX 2 410
   1      10  13   4   1                                       EX 2 420
   1      11   4   2   2                                       EX 2 430
   1      11  15   4   1                                       EX 2 440
   1      12   5   3   2                                       EX 2 450
   1      12  17   4   1                                       EX 2 460
   1      13  14   4   1     1                                 EX 2 470
   1      15  16   4   1     2                                 EX 2 480
   1      17  18   4   1     3                                 EX 2 490
   1      19   4   2   2                                       EX 2 500
   1      20   5   3   2                                       EX 2 510
   1      21   6   4   1                                       EX 2 520
          0                                                    EX 2 530
1  3  7                                                        EX 2 540
2  4  8                                                 6      EX 2 550
3  5  9                                                        EX 2 560
0                                                              EX 2 570
```

Figure 11. Input Data for Example 2

GERT SIMULATION PROJECT 2 BY SPACE EXPS
DATE 5/ 20/ 1970

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 21
NUMBER OF SOURCE NODES IS 1
NUMBER OF SINK NODES IS 1
NUMBER OF NODES TO REALIZE THE NETWORK IS 1
STATISTICS COLLECTED ON 10 NODES
NUMBER OF PARAMETER SETS IS 4
INITIAL RANDOM NUMBER IS 1267 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	P		A
4	1	1	P		A
5	1	1	P		A
6	1	9999	D		A
7	1	1	D		
8	1	1	D		
9	1	1	D		
10	1	1	D		
11	1	1	D		
12	1	1	D		
13	3	9999	D		
14	1	9999	D		F
15	3	9999	D		
16	1	9999	D		F
17	2	9999	D		
18	1	9999	D		F
19	1	1	D		A
20	1	1	D		A
21	1	1	D		A

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
6

STATISTICS COLLECTED ALSO ON NODES
21 20 19 18 16 14 5 4 3

Figure 12. Echo Check for Example 2

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS 1	2	3	4
1	10.0000	5.0000	20.0000	2.0000
2	20.0000	15.0000	25.0000	1.0000
3	15.0000	10.0000	30.0000	3.0000
4	0.0	0.0	0.0	0.0

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	2	0	0	1.0000
3	4	2	2	0	0	0.6000
3	10	4	1	0	0	0.3000
3	19	4	1	0	0	0.1000
4	5	3	2	0	0	0.5000
4	11	4	1	0	0	0.4000
4	20	4	1	0	0	0.1000
5	6	4	1	0	0	0.7000
5	12	4	1	0	0	0.2000
5	21	4	1	0	0	0.1000
7	4	2	2	0	0	1.0000
8	5	3	2	0	0	1.0000
9	6	4	1	0	0	1.0000
10	3	1	2	0	0	1.0000
10	13	4	1	0	0	1.0000
11	4	2	2	0	0	1.0000
11	15	4	1	0	0	1.0000
12	5	3	2	0	0	1.0000
12	17	4	1	0	0	1.0000
13	14	4	1	0	0	1.0000
15	16	4	1	0	1	1.0000
17	18	4	1	0	2	1.0000
19	4	2	2	0	3	1.0000
20	5	3	2	0	0	1.0000
21	6	4	1	0	0	1.0000

NETWORK MODIFICATIONS

ACTIVITY	NODE	FILE NODE	FILE NODE	FILE NODE	FILE NODE	FILE NODE	FILE NODE	FILE NODE	FILE NODE	FILE NODE
1	3	7								
2	4	8								
3	5	9								

Figure 13. Further Echo Check for Example 2

GERT SIMULATION PROJECT 2 BY SPACE EXPS
DATE 5/ 20/ 1970

FINAL RESULTS FOR 400 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
6	1.0000	66.1562	21.8904	400.	36.7050	137.6164	A
21	0.1300	68.2321	21.8631	52.	37.9771	118.8453	A
20	0.1525	44.6018	16.2177	61.	25.6203	96.1071	A
19	0.1575	13.3597	6.6827	63.	5.5631	32.2204	A
18	0.0350	77.9196	22.1003	14.	52.1680	119.6703	F
16	0.0675	73.4611	7.2378	27.	66.5126	93.7269	F
14	0.0350	29.9889	3.0599	14.	25.3156	36.5681	F
5	1.0000	65.8379	21.5760	493.	36.7050	137.6164	A
4	1.0000	46.1740	18.7907	670.	24.5228	114.1765	A
3	1.0000	14.1291	7.5545	576.	5.0000	43.2401	A

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
6	37.00	3.00	2	10	32	44	30	18	17	11	28	25	21
			17	18	12	11	14	19	13	7	4	5	2
			6	5	4	8	1	5	3	3	0	5	
21	41.00	3.00	4	5	1	5	2	2	0	5	2	0	3
			5	2	1	1	0	4	2	1	3	0	1
			0	1	0	0	2	0	0	0	0	0	
20	27.00	2.00	4	5	7	4	5	1	2	2	3	3	2
			2	2	4	2	1	0	0	0	3	3	0
			2	0	2	0	0	0	1	0	0	1	
19	1.00	1.00	0	0	0	0	0	2	3	4	7	8	10
			9	2	0	1	2	0	0	0	5	0	0
			2	3	0	0	0	2	0	1	0	2	
18	50.00	2.00	0	0	1	0	1	1	2	0	2	0	1
			0	0	0	0	0	0	0	1	0	1	1
			0	0	0	0	1	0	0	0	0	2	
16	65.00	1.00	0	0	4	1	1	2	6	2	4	1	0
			0	0	0	1	0	1	1	0	0	1	0
			0	0	0	0	0	0	1	1	0	0	
14	1.00	1.00	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0
			0	0	0	1	0	1	7	0	1	4	
5	37.00	3.00	2	12	39	49	41	24	21	15	36	35	27
			22	22	13	13	17	20	13	9	5	5	3
			10	5	6	9	1	6	4	4	0	5	
4	25.00	3.00	2	59	135	73	17	31	28	30	44	55	35
			8	20	15	15	14	23	10	4	8	8	4
			10	10	2	1	6	1	0	0	2	0	
3	5.00	1.00	0	11	20	41	59	74	81	55	37	13	11
			8	10	8	11	20	13	19	15	10	10	2
			1	4	10	3	2	3	5	2	3	15	

Figure 14. GERTS III Summary Report for Example 2

the histogram for node 4. Similar statistical quantities are available for the other nodes of the network.

Another interesting feature that could be incorporated into the network model for the sequencing of experiments is the changing of the sequence depending on the results of some of the experiments. This would be accomplished through the network modification procedures of the GERTS III program.

Table 1. Experiment Characteristics

Experiment	Probability of Success	Probability of Failure	Probability of Inconclusive Results	Allowable Numbers of Repeats
1	0.6	0.1	0.3	3
2	0.5	0.1	0.4	3
3	0.7	0.1	0.2	2

Experiment	Mean Time	Minimum Time	Maximum Time	Standard Deviation
1	10.0	5.0	20.0	2.0
2	20.0	15.0	25.0	1.0
3	15.0	10.0	30.0	3.0